

misleading: In decomposition theory random variables and vectors figure only in terms of their laws, and the theory, while its origin is probabilistic, is purely analytical. However, those defects—at least in the eyes of the reviewer—are of very little importance. For the book ought to be considered as a classic—the best of its kind. It is well written and very instructive.

The total impression about the state of the theory is somewhat disturbing. The ingenuity and power of the methods and the great wealth of results still leave the basic problem unsolved: Find applicable general criteria so that, given a law one can find all its components, and, in particular, find whether it is an indecomposable or an I_0 -law. It is hoped that the Linnik-Ostrovskiĭ book will serve as a catalyst for further search in this direction.

The untimely death of Linnik was a great loss for mathematics and for those who knew him.

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Completeness and basis properties of sets of special functions, by J. R. Higgins, Cambridge Tracts in Mathematics, no. 72, Cambridge Univ. Press, Cambridge, London, New York, Melbourne, 1977, x + 134 pp., \$19.95.

The questions considered in this book arise from our wanting to represent a given function as a linear combination of particularly interesting or useful auxiliary functions—for example, the eigenfunctions of a boundary value problem. In this setting the idea has been traced back to Daniel Bernoulli, who used the expansion as a formal device; it was Fourier who showed that (sometimes) the formal solution is really a solution. There are natural questions to ask about Fourier series (apart from their use in solving eigenvalue problems): Does the series converge? Does it converge to the function we got it from? If so, is it the only series of its kind that represents that function? A collection of functions φ_n such that every function f (in a suitable class) has a unique expansion $\sum a_n \varphi_n$ that converges (in a suitable topology) to f is called a basis. This notion, when formulated in abstract terms, can be considered in any Banach space, or even in more general spaces; a given set $\{\varphi_n\}$, regarded as abstract elements, may or may not form a base depending on which space they are taken to belong to. Thus for example the trigonometric functions $\{e^{inx}\}$ form a basis in L^2 (periodic functions of integrable square) but not in C (continuous functions under uniform convergence). The trigonometric functions also form an orthogonal set, but this is only a feature that is convenient for computing the coefficients in the expansion, not an essential part of the idea of a basis. Most of the familiar separable Banach spaces turn out to have bases, but we know (only since 1973) that there are separable Banach spaces that have no bases [5].

A similar idea entered mathematics in a different way and beginners sometimes confuse it with the idea of a basis. In abstract terms, a set $\{\varphi_n\}$ of elements of a Banach space is called total if every element of the space can be represented as the limit of a sequence of finite linear combinations of the φ_n , i.e. as $\lim_{N \rightarrow \infty} \sum_1^N a(k, N) \varphi_k$ rather than as $\lim_{N \rightarrow \infty} \sum_1^N a(k) \varphi_k$. This is the