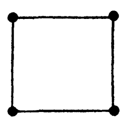
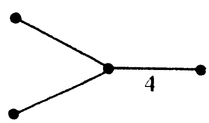
BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 84, Number 3, May 1978

Three-dimensional nets and polyhedra, by A. F. Wells, Wiley, New York, 1977, xii + 268 pp., $29.95.

In a storage room of the University of Minnesota, there used to be (and probably still are) four large isosceles triangular mirrors, with edges proportional to $2 : \sqrt{3} : \sqrt{3}$, relics of an abandoned film project. If they were put together as faces of an 'isosceles' tetrahedron (with some device to prevent the sloping mirrors from sagging under their own weight), and if you could look in through a hole in one of the edges, you would see a remarkable array of images. For this *tetragonal disphenoid* is one of the three kinds of tetrahedron that can serve as a fundamental region for a reflection group [Coxeter 1973, p. 84; Shubnikov and Koptsik 1974, p. 201]. It (or the group) is denoted by a 'Dynkin symbol'



in which the four dots represent the four mirrors while the four links indicate dihedral angles $\pi/3$ between pairs of mirrors. The mirrors represented by 'opposite' dots are at right angles because those dots are not directly linked. A plane that bisects one of these two right angles dissects the disphenoid into two congruent pieces, each of which is a *trirectangular* tetrahedron



The link marked 4 indicates that the cutting plane, which we naturally replace by a mirror, forms a dihedral angle $\pi/4$ with its neighbor. (Instead of a link marked 4, Witt [1941, p. 301] prefers a double link.) This smaller tetrahedron still has a plane of symmetry, bisecting one of its right angles, and this can be used to dissect the tetrahedron into two enantiomorphous pieces, each of which is an *orthoscheme*

This is the remaining kind of 4-mirror kaleidoscope. It is marked *CMIO* in the reviewer's Fig. 2.2A [Coxeter 1974, p. 13]. It combines with its image in the vertical plane *MIO* to form the trirectangular tetrahedron *CBIO*. This, in turn, combines with its image in the horizontal plane *CBI* to form the disphenoid *CBOO'*, which is where this discussion began.

By inserting rings round one or more of the dots, we symbolize a 'uniform honeycomb' whose vertices are all the images of a point that lies on all the 'unringed' mirrors, in a position equidistant from all the 'ringed' mirrors [Coxeter 1940, pp. 390–404]. In particular, if only one dot is ringed, the chosen point is one vertex of the tetrahedron; and if all four dots are ringed,