

point not on X lying on a block with each pair of the three points. (Picture a tetrahedron.) Then each point y not in X produces a partition of X via the set of associated “tetrahedra”, and by varying y a parallelism of X results (with $t = 3$). There are only two known examples of such biplanes $B(k)$, with $k = 3$ or 6 . A beautiful result of the author is presented: if, in addition, every four points of X lie in a subplane $B(6)$, then the biplane can only be $B(6)$. This is proved by a brief argument, in which it is shown that the parallelogram property must hold, and hence that the parallelism is of known type. Additional questions involving parallelisms and biplanes lead to some very special association schemes and metrically regular graphs.

Lurking in the background throughout many of these topics are the group theoretic situations in which many of the combinatorial questions were originally asked. This connection is described in the next to the last chapter. Consider a parallelism of t -sets of X , and let G be its automorphism group. If G is $(t + 1)$ -transitive on X and $|X| > 2t > 2$, then the parallelism is shown to have the parallelogram property (and hence is known) or to be of a unique type with $|X| = 6$, $t = 2$. This result, and its proof, are very typical examples of how large groups can be used in combinatorial situations: use of various stabilizers of one or more points of X leads to local “configuration” properties, which in turn permit purely combinatorial classification theorems to be applied. All the group-theoretic background required is proved (in yet another appendix); moreover, the group-theoretic question which required the preceding theorem is also presented. The discussion of automorphism groups concludes with a brief sketch of the classification of parallelisms for which G acts 2-transitively on the set of parallel classes.

Many open problems are presented throughout the book; indeed, the impression is clearly conveyed that any theorem, however beautiful and complete, easily leads to many problems. The final chapter discusses generalizations of the concept of parallelism. Naturally, large numbers of additional open problems result.

This book is a delight to read. The proofs are slick, but well motivated. It is short and carefully organized. The required background is minimal, being only part of a standard first year graduate algebra course. It would be an excellent way for a graduate student to learn many different techniques, some of which may be difficult, but all of which have clear cut and immediate applications to the main topic being studied.

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Lectures in semigroups, by M. Petrich, Akademie-Verlag, Berlin, 1977, viii + 168 pp.

What should a book on semigroups be about? Semigroups; of course, but the subject has long outgrown such a simple answer. In 1961 it was possible for Clifford and Preston [3] to attempt to be comprehensive, provided they stuck very strictly to the algebraic theory and ignored ordered and topological semigroups altogether; but by the time their second volume appeared in 1967