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*Estimates for the  $\bar{\partial}$ -Neumann problem*, by P. C. Greiner and E. M. Stein, Princeton Univ. Press, Princeton, N. J., 1977, 194 pp., \$6.00.

Several complex variables has enjoyed a renaissance in the past twenty-five years, reaching deeply into modern algebra, topology, and analysis for techniques to attack long standing problems. An important example is the question of identifying domains of holomorphy, i.e. those open sets in  $\mathbb{C}^{n+1}$  (or, more generally, in complex manifolds) for which at least one holomorphic function has no extension outside the set. Early in this century E. E. Levi defined a condition, now called *pseudoconvexity*, which he proved was necessary, and conjectured was sufficient, to characterise domains of holomorphy. More precisely, for domains with smooth boundary one can define a Hermitian form, now called the *Levi form*, on the space of holomorphic vectors tangent to the boundary. The domain is then called *pseudoconvex* (resp. *strictly pseudoconvex*) if the Levi form is positive semidefinite (resp. definite).

Levi's conjecture for  $\mathbb{C}^{n+1}$  was finally proved nearly fifty years later by Oka [16] (and simultaneously by Bremermann, [1] and Norguet [15]) after a long series of related papers. Efforts to extend the results to complex manifolds led Grauert [5] to discover a new, more general proof making extensive use of sheaf theory. A totally different proof was later obtained by Kohn [11] (using a crucial estimate of Morrey [13]) as a consequence of his solution of the " $\bar{\partial}$ -Neumann" boundary value problem in partial differential equations.

Since Kohn's breakthrough on the problem there has been considerable interest in constructing solutions for the inhomogeneous Cauchy-Riemann (C-R) equations in a bounded complex domain and studying their boundary behavior. Kohn's methods, based on a priori  $L^2$  estimates, give only  $L^2$  existence proofs for solutions of the C-R equations. (After Kohn's work appeared Hörmander [9] gave a simpler existence proof, using weighted  $L^2$  estimates, in which boundary problems are completely circumvented!) Several explicit solutions have been constructed by the use of integral formulas, in particular, those of Henkin [8] and Ramirez [18]. Kerzman [10], Grauert and Lieb [6], Overlid [17], and others have obtained estimates for these solutions in terms of  $L^p$  and Lipschitz norms.

Recently Greiner and Stein were able to give an explicit construction of Kohn's solution and to obtain from this construction optimal estimates in  $L^p$  and other norms. The book under review is an exposition of this work,