

THE PURE PHASES (HARMONIC FUNCTIONS)  
OF GENERALIZED PROCESSES<sup>1</sup>  
OR: MATHEMATICAL PHYSICS OF PHASE TRANSITIONS  
AND SYMMETRY BREAKING

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- I. Introduction. (Description of the problem, symmetries and broken symmetries).
- II. Lattice Systems and Generalized Processes. (The mathematical structure, interactions, finite systems, thermodynamic functions, equilibrium states, uniqueness theorems).
- III. The General Notion of Phase Transitions. (Definition of phase transitions, strategy for proving the existence of a phase transition, connection with symmetry breaking).
- IV. Reflection Positivity. (The cone of "reflection positive interactions", a generalization of the Hölder inequality for traces and the "chessboard estimates", examples of reflection positive interactions, *infrared bounds*).
- V. Application to Classical Lattice Systems: Phase Transitions for Gibbs Random Fields. (Application of results of §§III and IV to the proof of existence of phase transitions for a large class of systems, conclusions).

**I. Introduction.** This paper is written for mathematicians and mathematical physicists with some knowledge of stochastic processes and of the basic notions of statistical mechanics, but I have tried to explain what I believe are all major concepts, notions and definitions required for the understanding of the main results, i.e. I have tried to write these notes for the nonexpert at the risk of boring the expert and, perhaps, being a little imprecise here and there. (The expert may find some new results in §§IV and V.) *All* major recent or new results I am describing in this paper were obtained in collaboration with B. Simon, T. Spencer, E. H. Lieb and R. Israel. The reader is advised to consult references [1]–[4] for statements of the original results and complete proofs. Reviews of some of the material contained in these references and applications to relativistic quantum field theory may be found in [5], [6]. The reader may consult [7], [8], [1] for the original results on phase transitions in

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