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ASYMPTOTIC COMPLETENESS FOR A CLASS OF FOUR PARTICLE SCHRÖDINGER OPERATORS[1]

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1. Introduction. The purpose of this announcement is to state some new results in multichannel nonrelativistic quantum scattering theory.

The scattering theory of two particle nonrelativistic quantum mechanics is reasonably well understood for potentials which decrease at least as fast as $r^{-1-\epsilon}$ at infinity (see Reed and Simon [5] and the references therein). In comparison, relatively little is known about the general N particle problem, which for $N \geqslant 3$, involves multichannel scattering.

Asymptotic completeness and the absence of singular continuous spectrum in the three particle problem were first proved for a large class of two-body potentials by Faddeev [2]. Balslev and Combes [1] have proved the absence of singular continuous spectrum for N particles when the potentials are dilation analytic. However, no general asymptotic completeness results have previously been proved for $N \geqslant 4$, although Ginibre and Moulin [3], Thomas [9], and Howland [4] have simplified and extended Faddeev's asymptotic completeness results for $N = 3$.

Independent of the work being announced here, Sigal [7] has proved results which overlap with those stated below.

2. Main results. Let $\widetilde{H} = -\Sigma_{i=1}^4 \Delta_i/2m_i + \Sigma_{i<j} \lambda_{ij} V_{ij}$ be the Schrödinger operator for a system of four particles moving in $m \geqslant 3$ dimensions, and let $H = H_0 + \Sigma_{i<j} \lambda_{ij} V_{ij}$ denote the Schrödinger operator on $L^2(\mathbf{R}^{3m})$ for the same system with the center of mass motion removed.

For each pair i, j, $L^2(\mathbf{R}^{3m})$ decomposes into $L^2(\mathbf{R}^m) \otimes L^2(\mathbf{R}^{2m})$, where the first factor denotes functions of $x_{ij} = x_i - x_j$. Under this decomposition, $V_{ij} = v_{ij} \otimes 1$. It is assumed that $v_{ij} = u_{ij} w_{ij}$, such that both $u_{ij}(-\Delta + 1)^{-1/2}$ and $w_{ij}(-\Delta + 1)^{-1/2}$ are compact as operators on $L^2(\mathbf{R}^m)$ (here Δ denotes the Laplacian in the x_{ij} variable). U_{ij} and W_{ij} denote $u_{ij} \otimes 1$ and $w_{ij} \otimes 1$, respectively.

THEOREM. *Let $m \geqslant 3$, and suppose the potentials V_{ij} have been chosen so that*

(i) *each u_{ij} and w_{ij} is dilation analytic in some strip.*

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