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# ON QUANTUM MECHANICS OF MANY-BODY SYSTEMS WITH DILATION-ANALYTIC POTENTIALS

BY I. M. SIGAL

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1. In the paper [2] we have proved the asymptotic completeness of many particle nonrelativistic quantum systems with potentials, $V_\alpha$, satisfying the direct restriction

$$|v_\alpha(k)| + |h|^{-\nu}|v_\alpha(k+h) - v_\alpha(k)| \leq C(1 + |k|)^{-\eta}, \nu > \tfrac{1}{2}, \eta > \tfrac{3}{2},$$

where $v_\alpha(k) = \int V_\alpha(x)e^{-ikx}\,dx$, and the two indirect restrictions:

(i) No compound system has eigenvalues embedded in its continuous spectrum;

(ii) No compound system has quasibound states (at its thresholds). The definitions of a compound system and a quasibound state will be given later. The indirect restrictions are discussed in [2] and it has been stated there without proof that they are satisfied for "almost all" dilation analytic short range (analytic short range potentials) potentials. The aim of this note is to present the precise statements of the results mentioned above and some ideas of their proofs. We include also some other related results. Complete proofs will be published elsewhere.

2. Henceforth $H$ is the Hamiltonian of an $n$-body system in its center-of-mass frame.

THEOREM 1. *Let $V_{ij}$ satisfy the Combes conditions and besides let $V_{ij}(\theta)$ $\forall\theta \in 0$ obey*

$$\int |v_\alpha(k;\theta)|^m(1+|k|)^{\eta m}\,dk \leq C,\ m > 3,\ \eta > \tfrac{3}{2}\left(1 - \tfrac{2}{m}\right)$$

*Then the set of all $g \in \mathbf{R}^{n(n-1)/2}$ such that $H(g) = H_0 + \Sigma_{i<j} g_{ij}V_{ij}$ has no quasibound states is nondense in $\mathbf{R}^{n(n-1)/2}$.*

THEOREM 2. *Let $V_{ij}$ satisfy the assumptions of Theorem 1 and let no compound system have quasibound states (at its thresholds). Then the number of bound states of $H$ is finite and the resonances of $(H, D(O))$[1] have no accumulation points on the real axis.*

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[1]For the definition of the set $D(0)$ see [1]. The resonances of $(H, D(0))$ are nonreal eigenvalues of the Combes-Balslev family $H(\theta)$ ([1]).