10. K. Knopp, *Problem book in the theory of functions* II, Dover, New York, 1952, 138 pp. [A good collection of interesting exercises and problems for second and third semester courses in classical function theory. (Vol. I is rather elementary.)]

11. J. G. Krzyz, *Problems in complex variable theory*, American Elsevier, New York, 1971, xix + 283 pp. [For supplementing a regular course. Mostly exercises, but some real problems.]

12. Ya. I. Rivkind, *Problems in mathematical analysis*, Noordhoff, Groningen, ca 1965, v + 98 pp. [Meant to supplement real variable courses; mostly routine on the hard side.]

13. D. O. Shklarsky, N. N. Chentzov, and I. M. Yaglom, *The USSR Olympiad problem book*, Freeman, San Francisco, Calif., 1962, xvi + 452 pp. [An outstanding collection of problems on elementary mathematics.]

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15. G. Szász et al., *Contests in higher mathematics, Hungary* 1949–1961, Akadémiai Kiadó, Budapest, 1968, 260 pp. [A collection of highest quality.]

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HARLEY FLANDERS

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Rekursive Funktionen in der Komputer Theorie, by Rózsa Péter, Akadémiai Kiadó, Budapest, Hungary, 1976, 190 pp., $12.00.

The Theory of Recursive Functions developed in its present form in the decades following 1930. Pioneered by the work of Turing, Post and Church, it has aimed at making precise and at studying the notions of algorithm and computation.

A (partial) function from the set of natural numbers into natural numbers is *recursive* if it can be represented by an expression formed from certain *base functions* and the operations of *substitution*, *primitive recursion*, and *minimization*. The base functions comprise the *successor function* ($S(x) = x + 1$), the *null function* ($N(x) = 0$), and *projection functions* ($U_i^n(x_1, \ldots, x_n) = x_i$, where $1 \leqslant i \leqslant n$). Primitive recursion is used to define a function $h(z, x_1, \ldots, x_n)$ from recursive functions $f(x_1, \ldots, x_n)$ and $g(z, y, x_1, \ldots, x_n)$ by the pair of equations

$$h(0, x_1, \ldots, x_n) = f(x_1, \ldots, x_n),$$

$$h(S(z), x_1, \ldots, x_n) = g(z, h(z, x_1, \ldots, x_n), x_1, \ldots, x_n).$$

The operation of minimization defines a (possibly partial) function $f(x_1, \ldots, x_n)$ from a total recursive function $g(y, x_1, \ldots, x_n)$ as the "smallest y such that $g(y, x_1, \ldots, x_n) = 0$," and is written

$$f(x_1, \ldots, x_n) = (\mu y)\left[g(y, x_1, \ldots, x_n) = 0 \right].$$

Note that all recursive expressions can be enumerated and, hence, all recursive functions.

A. Church conjectured in 1936 that this class of functions was precisely the class of all effectively computable functions [**1**]. More accurately, to every effective rule for computing a sequence of natural numbers there exists a recursive expression with number e such that the function defined by the rule