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TWO PROOFS OF THE STABLE ADAMS CONJECTURE

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Let p and q be distinct primes. The complex Adams conjecture establishes a homotopy commutative diagram

$$\begin{array}{ccc} BU_{(p)} & \xrightarrow{\Psi^q} & BU_{(p)} \\ & {\scriptstyle J}\searrow \quad \swarrow{\scriptstyle J} & \\ & BsG_{(p)} & \end{array}$$

where J is the complex J-homomorphism and $\__{(p)}$ denotes localization at p. Both J and Ψ^q are infinite loop maps, and it is natural to ask whether this result is infinitely deloopable; that is, whether $J\Psi^q = J$ as infinite loop maps. This is the Stable Adams Conjecture.

We announce here two independent proofs of this conjecture. Details will appear in [2] and [6].

METHOD 1. Our proof is based upon a "geometric" criterion for pairs of maps into the spectrum $(\mathbf{BsG})^\wedge \cong (\mathbf{BsG})_{(p)}$ to be homotopic, where $(\ )^\wedge$ denotes the Bousfield-Kan $\mathbf{Z}/p$-completion functor. We exploit the "galois symmetry" of $(\mathbf{kU})^\wedge$ [8] to show that $J^\wedge, J^\wedge \circ (\Psi^q)^\wedge$ satisfy this criterion.

We impose a Quillen closed model category structure on Segal's Γ-spaces [3], whose weak equivalences are level-wise weak equivalences of spaces. For any "suitably oriented, pointed C. W.-like space" X (e.g., X any pointed C. W. complex with no orientation specified), we obtain a Γ-space $\mathcal{B}sG_X$ arising from distinguished homotopy equivalences of iterated smash products of X with itself. There is a natural functor

$$\Phi\colon \text{Ho } \Gamma\text{-}spaces \longrightarrow \text{Ho}Spectra$$

sending $\mathcal{B}sG_{S^2}$ to $\Phi(\mathcal{B}sG_{S^2}) = \mathbf{BsG}$.

Our basic representability theorem is a description of the functor

$$\text{Hom}_{\text{Ho}\Gamma\text{-spaces}}(\ \ ,\mathcal{B}sG_X)$$

as being isomorphic to the functor $sX(\ )$ of "oriented X-structures" over a variable Γ-space as base. For sufficiently nice X (e.g., $X = S^2$), $\text{Hom}_{\text{Ho}\Gamma\text{-spaces}}(\ \ ,(\mathcal{B}sG_X)^\wedge)$ is isomorphic to $\mathbf{Z}/psX(\ )$ (the theory of "oriented, $\mathbf{Z}/p$-completed X-structures"). The critical property of these X-structures is that they admit a functorial principal-

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