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RESEARCH ANNOUNCEMENTS

NONCOMMUTATIVE ERGODIC THEOREMS

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In [4], [5] E. C. Lance extended the pointwise ergodic theorem to actions of the group of integers on von Neumann algebras. Our purpose is to extend other pointwise ergodic theorems to von Neumann algebra context: the Dunford-Schwartz-Zygmund pointwise ergodic theorem (Theorem 3), the pointwise ergodic theorem for amenable locally compact connected groups (Theorem 4), the Wiener local ergodic theorem for $\mathbf{R}_+^d$ (Theorem 5) and for general Lie groups (Theorem 6).

We fix a pair $(\mathfrak{A}, \rho)$ where $\mathfrak{A}$ is a von Neumann algebra and ρ is a faithful normal state on $\mathfrak{A}$, we call *kernel* a positive linear contraction T of $\mathfrak{A}$ into itself such that $\rho(1) = 1$, $\rho(Ta) = \rho(a)$, $\rho((Ta)^*Ta) \leq \rho(a^*a)$ $\forall a \in \mathfrak{A}$. The set K of kernels of $\mathfrak{A}$ is a semigroup.

Let G be a locally compact semigroup, we call measurable (resp. continuous) representation of G into $\mathfrak{A}$ every (semigroup) homomorphism τ of G into K such that the mapping $G \ni g \longrightarrow \tau_g a \in \mathfrak{A}$ is ultrastrongly measurable (resp. continuous) for $a \in \mathfrak{A}$. The system $(\mathfrak{A}, \rho, G, \tau)$ is called a measurable (resp. continuous) *noncommutative dynamical system* (see [1], [5]).

In what follows, we assume that G admits a left invariant Haar measure ν. We call *amenable sequence* of G an increasing sequence $(V_n)_{n\geq 1}$ of Borel subsets of G such that $\bigcup_{n\geq 1} V_n$ generates G and

$$\sup_{g\in C} \frac{\nu(V_n \,\Delta\, gV_n)}{\nu(V_n)} \xrightarrow[n\to\infty]{} 0,$$

for every compact subset K' of C (cf. [3]).

We note $\mathfrak{A}^\tau = \{a \in \mathfrak{A} | \tau_g a = a \ \forall g \in G\}$.

Following E. C. Lance (cf. [4], [5]), we say that a sequence a_n of $\mathfrak{A}$ converges *almost everywhere* (or *almost uniformly*) to an element a of $\mathfrak{A}$ if for every $\epsilon > 0$, there exists a projection $e \in \mathfrak{A}$ such that $\rho(e) \geq 1 - \epsilon$ and $\|(a_n - a)e\| \longrightarrow 0$, $n \longrightarrow \infty$ (when $\mathfrak{A}$ is commutative, $\mathfrak{A} = L^\infty(X, \nu)$, with (X, ν) a probability space, this convergence coincides with the *almost everywhere pointwise convergence* via Egorov's theorem).

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