

“little more than a plaything”. The issue was forced by Bertrand’s paradox, which arose from the fact that the measure is not unique for geometric events depending on continuous parameters. Poincaré answered this question neatly by requiring the measures to be invariant under a group of transformations. In almost all practical cases this defines the measure up to a constant factor. Poincaré also realized the importance of the kinematic measure, which nowadays is better understood as the Haar measure of a unimodular Lie group, and exploited it.

The basic reason for integral geometry is the presence of a “duality” in most spaces. Examples are points and lines in the plane, points and geodesics in a Riemannian manifold, points and lattices in  $R^n$ , points and horospheres in a symmetric space, etc. The two dual geometric elements are related by a notion of incidence. Given a set in space, the measure of the set of dual elements incident to it gives an important invariant of the set. The classical and simplest example is Crofton’s theorem: The measure of the set of lines in the plane meeting an arc  $\beta$ , counted with multiplicities, is equal to twice the length of  $\beta$ . When the same idea is applied to lattices, it gives Siegel’s proof of Hlawka’s solution of a problem of Minkowski on convex bodies. Radon treated the problem of determining a function on the noneuclidean plane from the integrals of the function over all geodesics. This Radon transform was generalized by Fritz John, and later by Helgason, Gelfand, Graev, Vilenkin, and most recently by Guillemin. It plays an important role in partial differential equations and more general integral operators.

A natural application is to stereology, which deals with a body of methods for the exploration of three-dimensional space when only two-dimensional sections through solid bodies or their projections are available. Clearly stereology is useful in biology, mineralogy, and metallurgy. Recently the ideas and tools of stochastic processes are introduced, bringing the subject back to probability theory.

The strides made in the last four decades are enormous. Integral geometry is no longer a mathematical discipline to be ignored. But the subject has still the happy character that it is not so well known, thus allowing a steady and gentle progress.

As the first volume of an ambitious encyclopedia, the book sets a style. It is a combination of a lucid exposition of the introductory aspects and a complete survey of the area. No topic seems to have been left uncovered. There is also a very complete, but selective, bibliography. The author handled his material with great dexterity and ease. The book should serve as an excellent text for a graduate course on integral geometry. The encyclopedia and the author are to be congratulated for their success.

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*Methods of accelerated convergence in nonlinear mechanics*, by N. N. Bogoljubov, Ju. A. Mitropoliskii and A. M. Samoilenko, Hindustan Publishing Corporation, Delhi, India, viii + 291 pp., \$27.90.

(I) The averaging method—or what is so called today—has its origin in