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TORSION IN THE HOMOLOGY OF H-SPACES

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The purpose of this note is to announce some consequences of lack of torsion in $H_*(\Omega X; \mathbf{Z})$ when (X, μ) is a 1-connected H-space of finite type. Using this hypothesis we can deduce certain restrictions on the occurrence of torsion in the ordinary homology of X as well as in its BP, MU, and K homology. Our motivation for this approach comes from finite H-space theory. Certain cases of our restrictions or of the absence of torsion in $H_*(\Omega X; \mathbf{Z})$ have been proven for finite H-spaces (see [6]) or, at least, for compact Lie groups (see [1], [3] and [7]). Our arguments tie these results together and, furthermore, show that the relations do not depend on the finiteness of the spaces involved.

For the rest of the paper let p be a fixed prime and Q_p the integers localized at p. Let $H_*(X) = H_*(X; \mathbf{Z}) \otimes_{\mathbf{Z}} Q_p$. Let (X, μ) be a 1-connected H-space of finite type such that $H_*(\Omega X)$ is torsion free.

THEOREM 1. *$H_*(X)$ has no higher p torsion.*

Now $BP_*(X)$ is a module over

$$\Lambda = BP_*(pt) = Q_p[v_1, v_2, \ldots] \ (\deg v_s = 2p^s - 2).$$

Thus, besides p torsion, we can also speak of v_s torsion for $s \geqslant 1$. However, the various torsion submodules are interrelated. In particular they are all contained in the v_1 torsion submodule. For let $\Lambda(1) = \Lambda(1/v_1)$ and $BP_*(X; \Lambda(1)) = BP_*(X) \otimes_\Lambda \Lambda(1)$.

THEOREM 2. *$BP_*(X; \Lambda(1))$ is torsion free.*

We can also deduce results about the algebra structure of $BP_*(X; \Lambda(1))$. Let P and Q denote primitives and indecomposables respectively.

THEOREM 3. *$BP_*(X; \Lambda(1))$ is commutative (associative) if, and only if, $H_*(X) \otimes_{\mathbf{Z}} Q$ is commutative (associative). When $H^*(X) \otimes_{\mathbf{Z}} Q$ is an exterior algebra then $BP_*(X; \Lambda(1))$ is generated as an algebra by the image of the delooping map $\Omega_*\colon Q(BP_*(\Omega X; \Lambda(1))) \to P(BP_*(X; \Lambda(1)))$.*

Furthermore, it is necessary to localize with respect to v_1 to obtain these types of results.

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