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REPRESENTING HOMOLOGY CLASSES BY EMBEDDED CIRCLES AND THE EXISTENCE OF CIRCLES INVARIANT UNDER ISOMETRIES

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ABSTRACT. Paper concerns the problem of representing homology classes by embedded circles, and the question of existence of circles invariant under an isometry of a compact surface.

If $I: \mathbf{R}^3 \longrightarrow \mathbf{R}^3$ is an isometry and $M \subset \mathbf{R}^3$ is an embedded compact invariant surface, then we can prove that there is always a circle on M which is invariant under I. This result follows from Theorem 3 and the fact that any isometry of the sphere or torus has an invariant circle.

Let $f: M \longrightarrow M$ denote an orientation preserving diffeomorphism of finite order on a compact oriented surface, and let $P: M \longrightarrow M_f$ be the natural projection to the orbit or quotient space M_f. We will consider two embedded circles to be equivalent if they are isotopic through invariant circles.

THEOREM 1. *Let $f: M \longrightarrow M$ where $M \neq S^2$. Then*

(1) *There exist an infinite number of distinct homology classes represented by an invariant circle iff $M_f \neq S^2$ or $f^2 = \mathrm{id}_M$.*

(2) *If $M_f = S^2$ and $f^2 \neq \mathrm{id}_M$, each invariant circle disconnects M.*

THEOREM 2. *There exists an $f: M \longrightarrow M$ of order* 30 *on a surface of genus* 11 *with the following properties.*

(1) *f has no invariant circle.*

(2) *If $g: M \longrightarrow M$ has no invariant circles then g is conjugate to f^r where r is relatively prime to* 30.

THEOREM 3. (1) *If $f: M \longrightarrow M$ has order $p^k q^l$ where p and q are primes, then f has an invariant circle.*

(2) *If the genus of M is less than* 11 *then every $f: M \longrightarrow M$ has an invariant circle.*

(3) *If $f: M \longrightarrow M$ is induced by an isometry $F: \mathbf{R}^3 \longrightarrow \mathbf{R}^3$ then f has at least* 4 *invariant circles when the genus of M is greater than* 1.

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