

ULTRAFILTERS: SOME OLD AND SOME NEW RESULTS

BY W. W. COMFORT¹

I am grateful to Peter Freyd for his generous introductory comments, and I am grateful also to the [Selection] Committee for extending the invitation to speak to you.

Last month a colleague with whom I attempted to discuss today's remarks suggested drily that as half of the team responsible for [CN2] I have probably already said more than enough about ultrafilters, and that if I insist on pursuing the matter further today I could do so most gracefully and efficiently simply by offering a complimentary copy of [CN2] to each of you. Eschewing that advice I shall in the hour allotted to me attempt to achieve the following three goals.

(A) To acquaint you with what I think are some of the most basic, fundamental facts about ultrafilters on a discrete topological space; this material is sufficiently simple and elegant that it can be absorbed comfortably into a first-year graduate course in general topology.

(B) To give some partial results, less definitive and less conclusive than the optimal theorems available, concerning the existence of particular ultrafilters with special properties; I hope that the results chosen in this connection have the complementary virtues that they are sufficiently powerful to handle most of the situations treated by the more powerful results which we shall ignore, and that their proofs are significantly simpler than those of the more general results.

(C) To record some results about ultrafilters which came to my attention after the publication of [CN2]; I have chosen today to emphasize three relatively new results which are not formally concerned with ultrafilters and which indeed make no mention of ultrafilters in their statements, but which nevertheless have been given proofs in which ultrafilters play an important catalytic role.

My hope is that (even) those of you not professionally inclined toward topology or set theory will find something potentially useful, or amusing, among the basic results given in (A). The theorems selected for inclusion in (B) are given not only because of the intrinsic beauty and elegance of their proofs, but also because they serve to indicate the principal sorts of questions

An address delivered at the Eastern Sectional Meeting of the American Mathematical Society in New York, New York on April 13, 1976; received by the editors August 15, 1976.

AMS (MOS) subject classifications (1970). Primary 04A20, 04A30, 10A45, 54A25, 54C45, 54D20, 54G05.

¹ The author gratefully acknowledges partial support received from the National Science Foundation under grants MPS73-08937A02 and MCS76-05821.