BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 83, Number 2, March 1977

PARTS OF MEASURES AND INTEGER-VALUED TRANSFORMS

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Communicated by Richard R. Goldberg, September 20, 1976

In this paper G is a compact abelian group with ordered dual Γ. By this we mean there is a nontrivial group homomorphism $\phi: \Gamma \longrightarrow \mathbf{R}$ where $\mathbf{R}$ is the additive group of real numbers. Let $M(G)$ be the usual convolution algebra of finite Borel measures on G and $\hat{}$ the Fourier-Stieltjes transformation.

A measure $\mu \in M(G)$ is said to vanish at infinity in the direction of ϕ if $\{\gamma_n\}_1^\infty \subset \Gamma$ with $\phi(\gamma_n) \longrightarrow \infty \Rightarrow \hat{\mu}(\gamma_n) \longrightarrow 0$. The subspace consisting of all measures whose transforms vanish at infinity in the direction of ϕ will be denoted by $M_\phi(G)$.

Let δ_0 be the identity measure in $M(G)$ and for any integer N_i put $\delta_i = N_i\delta_0$. The purpose of this note is to announce the following results which explicate a line of research begun by H. Helson [2] and continued by various authors in [1], [3], [5], [6], and [7].

THEOREM 1. *Let $\mu \in M(G)$ such that the convolution product $\Pi_{i=1}^m(\mu - \delta_i) \in M_\phi(G)$. Then μ has a decomposition $\mu = \mu_0 + \mu_\perp$ where $\mu_0 \in M_\phi(G)$, $\mu_\perp \in M_\phi^\perp(G)$ and $\hat{\mu}_\perp(\Gamma) \subset \{N_1, \ldots, N_m\}$. If $\Pi_{i=1}^m(\mu - \delta_i) \in M_0(G)$ then μ has a decomposition $\mu = \mu_0 + \mu_\perp$ where $\mu_0 \in M_0(G)$, $\mu_\perp \in M_0^\perp(G)$ and $\hat{\mu}_\perp(\Gamma) \subset \{N_1, \ldots, N_m\}$. Here $M_0(G)$ is the ideal of measures $\mu \in M(G)$ such that $\hat{\mu} \in C_0(\Gamma)$.*

The proof of Theorem 1 is obtained by analyzing $\mu_\perp$ in $M(\mathbf{S})$ where $\mathbf{S}$ is the structure semigroup of $M(G)$.

Assume ϕ is an isomorphism, P the positive cone and $\mathbf{E}$ a Sidon subset of Γ. For any subset A of Γ put $\mathbf{F}(A) = \{\mu \in M(G): \hat{\mu}$ is integer-valued on $A\}$ and $\mathbf{I}(A) = \{\mu \in M(G): \hat{\mu} = 0$ or 1 on $A\}$. The following theorem is a consequence of Theorem 1 and is an extension of a result announced by I. Kessler [3]; see also [4, pp. 206–211].

THEOREM 2. *If $\mu \in \mathbf{F}(\Gamma \backslash -P \cup \mathbf{E})$ then there is a $\nu \in \mathbf{F}(\Gamma)$ such that $\hat{\mu} = \hat{\nu}$ off $-P \cup \mathbf{E}$. In particular, if $\mu \in \mathbf{I}(\Gamma \backslash -P \cup \mathbf{E})$ then $\nu \in \mathbf{I}(\Gamma)$.*

Measures such that $\hat{\mu}(\gamma) = \hat{\mu}^2(\gamma)$ for all $\gamma \in P$ are called semi-idempotents. A subset $\mathfrak{R}$ of Γ is said to be a weak Rajchman set if supp $\hat{\mu} \subset \mathfrak{R} \Rightarrow \hat{\mu} \in C_0(\Gamma)$. An easy consequence of Theorem 1 is the following result.

AMS (MOS) subject classifications (1970). Primary 43A25.