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AUTOMORPHIC CUSP FORMS CONSTRUCTED FROM THE WEIL REPRESENTATION

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We recall the notation and results of [3].

Let $\mathbf{Q}$ be the rational numbers.

We let L be a Q integral lattice in $\mathbf{Q}^k$, i.e. $Q(\xi_1, \xi_2) \in \mathbf{Z}$ for all $\xi_1, \xi_2 \in L$. Let $L_*(Q)$ be the Q dual of L, i.e. $L_*(Q) = \{\eta \in \mathbf{R}^k \mid Q(\eta, \xi) \in \mathbf{Z}, \forall \xi \in L\}$. Then $L_*(Q)/L$ is a finite Abelian group, and we let N_L be the exponent of $L_*(Q)/L$, i.e. the smallest positive integer x so that $x \cdot \xi \in L$ for all $\xi \in L_*(Q)$. Choosing a $\mathbf{Z}$-basis X_i of L, we let $D_{Q(L)} = \det\{Q(X_i, X_j)\}$. Then the integer $D_{Q(L)}$ is independent of the choice of basis of L.

Then we define

$$\Gamma_L(Q) = \{g \in O(Q) \mid g(L) = L\}$$

and

$$\Gamma^L(Q) = \left\{ \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}, \epsilon \right) \middle| a, b, c, d \in \mathbf{Z}, ad - bc = 1, \right.$$
$$\left. b \equiv 0 \bmod 2 \text{ and } c \equiv 0 \bmod 2N_L \right\}.$$

Then $\Gamma_L(Q)$ is an arithmetic subgroup of $O(Q)$ and $\Gamma^L(Q)$/(cyclic group of order 4) is an arithmetic subgroup of $\mathrm{PSl}_2(\mathbf{R})$ (contained in the Γ_ϑ theta group). Then using the corollary to Theorem 5 of [3] we have

THEOREM 1. *Let φ be a $\widetilde{K} \times K$ finite function in $\mathbf{F}_Q^+(s^2 - 2s)$ with $s > \frac{1}{2}k$. Then the sum with $(G, g) \in \widetilde{\mathrm{Sl}_2} \times O(Q)$,*

$$T_\varphi^L(G, g) = \sum_{\xi \in L} \pi_Q(G, g)^{-1}(\varphi)(\xi), \tag{1.1}$$

is absolutely convergent. Moreover, for $(\Omega, \gamma) \in \Gamma^L(Q) \times \Gamma_L(Q)$, we have the functional equation

$$T_\varphi^L(G\Omega, g\gamma) = \sigma_Q^L(\Omega, \gamma) T_\varphi^L(G, g), \tag{1.2}$$

where σ_Q^L is a unitary character on $\Gamma^L(Q) \times \Gamma_L(Q)$ taking values in S_4 (where $S_j = \{z \in \mathbf{C} \mid z^j = 1\}$ for j any positive integer). Moreover, T_φ^L is a C^∞ function on $\widetilde{\mathrm{Sl}_2} \times O(Q)$ satisfying $D * T_\varphi^L(G, g) = T_{\pi_Q(D)\varphi}^L(G, g)$ for any D in the

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