

contraction extension property in terms of the familiar Kirszbraun property (K).

$(X, Y)$  is said to have property (K), if, for any fixed set  $I$ , and any pair of families  $\{B(x_i, r_i): i \in I\}, \{B(y_i, r_i): i \in I\}$  of closed balls in  $X$  and  $Y$ , respectively, such that  $d_2(y_i, y_j) \leq d_1(x_i, x_j)$  ( $i, j \in I$ ),

$$\bigcap B(x_i, r_i) \neq \emptyset \Rightarrow \bigcap B(y_i, r_i) \neq \emptyset.$$

Property (K) is then shown to be equivalent to the contraction extension property. This fact is helpful in showing that for a Hilbert space  $H$ , the pair  $(H, H)$  has the extension property for Lipschitz-Hölder maps. (This result is generalized to pairs  $(L^p, L^q)$  in the closing pages of the book.) Moreover, within the class of strictly convex Banach spaces no other pair  $(X, X)$  has this property. A similar result, due to S. Schönbeck, holds for pairs  $(X, Y)$  where  $Y$  is strictly convex and  $\dim Y \geq 2$ , though the proof of this fact is considerably more involved. Without strict convexity of  $Y$  the problem is, in general, rather difficult and partial solutions are, therefore, of interest. One of these due to B. Grünbaum (for  $\dim X = 2$ ) and to S. Schönbeck, states that if  $X$  is a separable conjugate Banach space, then for  $(X, X)$  to have the contraction extension property it must be a Hilbert space or have the binary intersection property for closed balls. Along with property (K) and the above mentioned property, other intersection theorems for families of closed balls are known to be useful when dealing with the extension of contractions, and some of these are presented in that context. Briefly touched upon are the investigations of D. de Figueiredo and L. Karlovitz into the existence of contractive retractions over a closed convex subset of a Banach space, as well as those of F. Valentine on the contraction extension property of  $(X, X)$  when  $X$  is an  $n$ -sphere. In a departure from the main theme several other loosely connected topics are dealt with. Among these are the extension problem for uniformly continuous mappings, and, in a different direction altogether, a packing problem for the unit ball in  $L^p$ .

For a slim volume, about a hundred pages long, the amount of material covered is considerable, and while the authors may have omitted some topics which are relevant to the subject matter, and included others which are less so, the balance seems satisfactory. The writing is exceedingly clear and the pace easy to keep up with.

This book should help to arouse a more widespread interest in an area in which the interplay between geometry and analysis is both fruitful and pleasing. As such, it is most welcome.

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*A geometric approach to homology theory*, by S. Buoncrisiano, C. P. Rourke, and B. J. Sanderson, London Mathematical Society Lecture Note Series, no. 18, Cambridge Univ. Press, New York and London, 1976, 149 pp., \$10.95.