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BOOK REVIEWS

Large infinitary languages, model theory, by M. A. Dickmann, Studies in Logic and the Foundations of Mathematics, Vol. 83, North Holland/American Elsevier, Amsterdam and Oxford/New York, xv + 464 pp., $39.95.

This book is a comprehensive compilation of model theoretic results concerning the languages $L_{\kappa\lambda}$. Most of the results have been previously published over the last 15 or 20 years, but they are carefully organized and rounded out in the book. Although containing a large amount of highly technical material, the book is surprisingly readable due to the informal remarks which keep the reader informed as to the objectives of and the methods used in the detailed presentation.

The languages, as well as the problems considered, are strongly involved with cardinality consideration. As defined by Tarski in a 1958 paper (with $\kappa = \lambda$), $L_{\kappa\lambda}$ is a first order language making use of the same models and atomic formulas as ordinary predicate logic (which can be identified with $L_{\omega\omega}$). However, in building up formulas, conjunctions of sets of formulas of power less than κ and quantifications over sets of variables of power less than λ are allowed. If κ is bigger than ω, this, of course, allows formulas which are not "writable" in the sense of being finite strings of symbols. Also, in the book, no notions of recursiveness, admissibility, or even definability are applied to the formulas or sets of formulas considered. Nevertheless, the purely model theoretic notions developed for $L_{\omega\omega}$ are easily applied to these languages and the results obtained turn out to be quite different.

The reader is expected to have a fairly broad background in logic and set theory. Some specialized topics in set theory that he is less likely to be familiar with are taken up in Chapter 0. One section takes up filters, ultraproducts and trees; another takes up the transition from countably additive real-valued measures on all subsets of a set to the existence of κ-additive two-valued measures on a cardinal κ; and still another takes up the partition calculus and the relation $m \to (n)^r_\kappa$. Of even greater importance are the sections on cardinal arithmetic and the Mahlo hierarchies of inaccessible cardinals. The interested reader should work out for himself, at each stage of the string of definitions, some further facts concerning the size of the cardinals being considered. For example, note how much bigger the gap is between the second and third inaccessible (if they exist) than between the first and second. Only then will he be equipped to judge for himself the importance of theorems in the remainder of the book concerning cardinals much larger than these and whose existence is much more conjectural.

Chapter 1 introduces the languages, their truth definition, and other basic model theoretic concepts. Some of the expressive power of the languages is revealed in examples. It is interesting to note that some concepts that are most naturally expressed in second order language (such as the concept of a well-ordered set or a simple group) can also be expressed by a denumerable first order formula. Other second order conditions (such as a set being the power set of another set and various completeness conditions with unrestricted