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ASPECTS OF VALUE DISTRIBUTION THEORY IN SEVERAL COMPLEX VARIABLES

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During the last fifty years value distribution in one complex variable has been established as one of the most beautiful branches of complex analysis. In several variables, value distribution was slow to grow up. Only a few people were concerned and many obstacles had to be overcome. However, recently, the theory has gained wide recognition. The outlook for the future is bright and promises a theory even broader in scope than its one-dimensional counterpart.

1. The classical theory. At first let us look at some basic results in one variable. Realize the Riemann sphere $\mathbf{P}_1$ as a sphere of diameter 1 in $\mathbf{R}^3$. The chordal distance between points w and a in $\mathbf{P}_1$ is denoted by $\|w, a\|$. Then $0 \leqslant \|w, a\| \leqslant 1$. The Riemann sphere carries a rotation invariant volume element Ω giving the sphere total volume 1. As on each complex manifold, the exterior derivative $d = \partial + \bar{\partial}$ twists to

$$d^c = (i/4\pi)(\bar{\partial} - \partial).$$

On $\mathbf{P}_1 - \{a\}$, the volume element Ω is computed by

$$\Omega = -dd^c \log\|a, w\|^2. \tag{1.1}$$

If $r > 0$, let $\mathbf{C}[r]$ be the closed disc, $\mathbf{C}(r)$ be the open disc and $\mathbf{C}\langle r\rangle$ be the circle, all of radius r and with center 0. Let $f: \mathbf{C} \to \mathbf{P}_1$ be a nonconstant holomorphic map, i.e., a nonconstant meromorphic function. The *spherical image* of f is defined by

$$A_f(r) = \int_{\mathbf{C}(r)} f^*(\Omega) > 0.$$

For $0 < s < r$, the *Ahlfors-Shimizu characteristic* of f is defined by

$$(T_f(r, s)) = \int_s^r A_f(t)\, dt/t.$$

Then $T_f(r, s) \to \infty$ for $r \to \infty$. On $\mathbf{C}\langle r\rangle$, a rotation invariant line element σ exists which gives the circle $\mathbf{C}\langle r\rangle$ length 1. For $r > 0$, the *compensation function* of f for $a \in \mathbf{P}_1$ is defined by

$$m_f(r, a) = \int_{\mathbf{C}\langle r\rangle} \log \frac{1}{\|f, a\|}\, \sigma \geqslant 0.$$

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