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DEFORMING HOMOTOPY EQUIVALENCES TO HOMEOMORPHISMS IN ASPHERICAL MANIFOLDS

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ABSTRACT. Aspherical manifolds are closed manifolds which are $K(\pi, 1)$'s. They play a significant role in many branches of mathematics. This paper constructs "model" aspherical manifolds for various classes of π and investigates and surveys their groups of homeomorphisms. It is not known whether aspherical manifolds having the same fundamental groups as our model manifolds can topologically differ from them. If π contains a normal abelian subgroup then the model aspherical manifolds are special instances of injective Seifert fiber spaces. The group of (singular) fiber preserving homeomorphisms of Seifert fiber spaces are characterized and criteria obtained so that each self-homotopy equivalence may be deformed to such a homeomorphism. Many of our model aspherical manifolds satisfy these criteria. Other applications are given to group theory, differential geometry, complex manifolds as well as topology. We have also included a list of unsolved problems.

1. Introduction. This article, which is an expanded version of an hour address by the second author, is largely, but not exclusively, devoted to the study of *aspherical* manifolds. These are **closed** manifolds M whose universal coverings are *contractible.* Thus, they are $K(\pi, 1)$'s where π is their fundamental group ($\pi = \pi_1(M)$). Aspherical manifolds occur quite naturally in several *complex variables* (e.g., Riemann surfaces, quotients of bounded domains, hyperbolic manifolds, holomorphic Seifert fiberings), *differential geometry* (e.g., manifolds whose sectional curvature is $\leqslant 0$), *Lie group theory* (e.g., double coset spaces $K\backslash G/\Gamma$, where G is connected, K a maximal compact and Γ a torsion free uniform subgroup), *group theory* (e.g., groups whose cohomological dimension is finite), *number theory* and *algebraic geometry* (e.g., "fiber varieties", Brieskorn varieties), and of course *topology.* (Specific contacts with these fields arise within this paper as either part of the theory or as illustrative examples.) This has led to their study by many diverse methods. Consequently, the exploration of topological properties and their topological identification and classification takes on added significance. We may ask which groups π

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