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CONTINUITY OF THE KOBAYASHI METRIC IN DEFORMATIONS AND FOR ALGEBRAIC MANIFOLDS OF GENERAL TYPE[1]

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Let M be a complex manifold and TM the holomorphic tangent bundle of M. The disc of radius r in $\mathbf{C}$ will be denoted by $\Delta(r)$, and Δ will stand for $\Delta(1)$. The Kobayashi pseudo-distance d_M and its infinitesimal pseudo-metric F_M are defined as follows:

(i) If $p, q \in M$, then

$$d_M(p, q) = \inf_{\{a_i\} \subset \Delta} \frac{1}{2} \sum_i \log \frac{1 + |a_i|}{1 - |a_i|}$$

where the infimum is over all finite sets $\{a_i\} \subset \Delta$ such that there exist n analytic mappings f_i: $\Delta \longrightarrow M$ for which $f_1(0) = p, f_i(a_i) = f_{i+1}(0)$ for $i = 1, n - 1$, and $f_n(a_n) = q$.

(ii) If $\langle x, \xi\rangle \in TM$, then $F_M(x, \xi) = \inf 1/R$ where the infimum is over all R such that there exists an analytic f: $\Delta(R) \longrightarrow M$ with $f_x(0, \partial/\partial z|_0) = \langle x, \xi\rangle$.

Royden has shown [5] that $d_M(p, q) = \inf_\sigma \int_\sigma F(\sigma, \dot\sigma)$ where the infimum is over all piecewise smooth curves from p to q.

The manifold M is said to *hyperbolic* if $d_M(p, q) \neq 0$ whenever $p \neq q$.

A deformation of M is specified by giving an analytic space $S \subset \mathbf{C}^k$ and a family of integrable almost complex structures $\{\varphi_s | s \in S\}$ on M such that $\varphi_o = 0$ for some point $o \in S$; each φ_s is therefore a C^∞ TM-valued (0, 1) form on M, satisfying $\bar\partial\varphi_s - [\varphi_s, \varphi_s]/2 = 0$. See [2] for details. Using φ_s, we can construct a bundle isomorphism Φ_s: $TM \longrightarrow TM_s$, where TM_s is the holomorphic tangent bundle for the complex structure given by φ_s. Set $F_{M_s} = F_s$. Assume that $o = 0$, the origin in $\mathbf{C}^k$.

THEOREM A. *Given $\langle x, \xi\rangle \in TM$ and $\epsilon > 0$, there exists a $\delta > 0$ such that if $|s| < \delta$ then $F_s(y, \eta) \leqslant F_o(x, \xi) + \epsilon\|\xi\|$ for all $\langle y, \eta\rangle$ in a neighborhood of $\langle x, \Phi_s\xi\rangle$ in TM_s. (Here $\|\xi\|$ is the norm provided by a coordinate system.)*

This basic upper semicontinuity result can be improved if F_M is known to

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