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COMPACTIFYING THE JACOBIAN

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Compactifications of jacobians of integral curves and their compatibility with specialization were considered by Igusa (Amer. J. Math. 1956) for members of a Lefschetz pencil, by Mayer-Mumford (Woods Hole, 1964) for any integral curve in terms of the moduli of torsion-free sheaves of rank 1, and by D'Souza (Bombay thesis, 1973, being improved for *Astérisque*) who proved the "Abel map" from the Hilbert scheme smooth where it should be for a Gorenstein curve and the compactification irreducible for a curve with nodes. Up to now, it has been impossible to make Grothendieck's fine theory (FGA) of the Picard scheme compactify the jacobian; we announce what we feel is a conceptual and technical breakthrough.

Let $f: X \longrightarrow S$ be a projective morphism of locally noetherian schemes. The heart of our theory is a good notion of the functor of linear systems: for I, F coherent on X with F S-flat, set

$$\operatorname{Lin\,Sys}_{I,F}(T) = \{0 \longrightarrow I(G) \longrightarrow F_T \longrightarrow G \longrightarrow 0 \in \operatorname{Quot}_{F/X/S}(T) | I(G) \simeq I_T \otimes N \text{ for some invertible sheaf } N \text{ on } T\}.$$

The functor is representable ((1) below) by a manageable scheme if I is S-*simple*, i.e., S-flat with $0_S \longrightarrow f_*\operatorname{Hom}(I, I)$ universally an isomorphism. (1) is easy to prove using (EGAIII$_2$, 7.7.9): There exists a coherent sheaf $H = H(I, F)$ on S and an element $h = h(I, F)$ of $\operatorname{Hom}_X(I, F \otimes H)$ such that for every quasi-coherent sheaf M on an S-scheme T the Yoneda map is an isomorphism

$$y(h_T): \operatorname{Hom}_T(H_T, M) \xrightarrow{\sim} \operatorname{Hom}_{X_T}(I_T, F_T \otimes M).$$

(1) THEOREM. *If I is S-simple, then* $\operatorname{Lin\,Sys}_{I,F}$ *is universally represented by an open subscheme U of $P = \mathbf{P}(H(I, F))$, and U is all of P if and only if, for every geometric point s of S, every nonzero map $I(s) \longrightarrow F(s)$ is actually injective.*

Assume f flat with geometric fibers integral curves of arithmetic genus p_a. Our form of the existence theorems are (2), (3) below. (3) is proved via g-rigidification. Set

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