BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 82, Number 6, November 1976

THE CONJUGATING REPRESENTATION OF A SEMISIMPLE ALGEBRAIC GROUP

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Communicated by Hyman Bass, July 12, 1976

Let G be a connected, simply connected semisimple algebraic group over an algebraically closed field k of characteristic zero and let $A(G)$ denote the algebra of regular functions on G. We let G act morphically on G by conjugation. This defines a representation of G on $A(G)$. In analogy with the terminology for finite groups we call this the conjugating representation of G. Let $C(G)$ be the algebra of all (regular) class functions on G; equivalently $C(G) = A(G)^G$. Let Λ be a set of representatives for the equivalence classes of irreducible rational representations of G. If $\lambda\colon G \longrightarrow \mathrm{GL}(V_\lambda)$ is an element of Λ, let $d_\lambda = \dim V_\lambda$ and let $m_\lambda = \dim V_\lambda^T$, where T is a maximal torus of G. Let $A(G)_\lambda$ denote the isotopic component of $A(G)$ of type λ. Each $A(G)_\lambda$ is a finitely generated $C(G)$-module and $A(G)$ is the $C(G)$-module direct sum of the $A(G)_\lambda$, $\lambda \in \Lambda$.

THEOREM A. *Let $\lambda \in \Lambda$. Then there exists a G-stable vector subspace H_λ of $A(G)_\lambda$ such that the k-linear map $C(G) \otimes_k H_\lambda \longrightarrow A(G)$ defined by $c \otimes h \longrightarrow ch$ is an isomorphism of vector spaces. The rational G-module H_λ is equivalent to the direct sum of m_λ copies of V_λ. In particular, $A(G)_\lambda$ is a free $C(G)$-module of rank $d_\lambda m_\lambda$ and $A(G)$ is a free $C(G)$-module.*

For the adjoint action of G on the Lie algebra $\mathfrak{g}$ and the corresponding representation of G on the algebra $A(\mathfrak{g})$ of polynomial functions on $\mathfrak{g}$, a similar theorem was proved by Kostant [1]. In the Lie algebra case, the proof of freeness is greatly simplified by the graded algebra structure of $A(\mathfrak{g})$. Our proof makes use of the results of Steinberg [4] (see also [3], [5]) on conjugacy classes in G and on the fibres of the morphism $\pi\colon G \longrightarrow k^r$, $r = \operatorname{rank} G$, given by the fundamental characters. We use the methods of commutative algebra. In particular, we require two separate applications of the Serre conjecture on projective modules over polynomial rings, which has recently been proved by Quillen [2].

If the base field k is of prime characteristic, we have a different proof that $A(G)$ is a free $C(G)$-module. In this case we have not been able to prove the existence of a *G-stable* vector subspace H of $A(G)$ such that the product map $C(G) \otimes_k H \longrightarrow A(G)$ is a vector space isomorphism.

In §1 we outline the proof of Theorem A. A detailed proof will appear elsewhere.

AMS (MOS) subject classifications (1970). Primary 20G05; Secondary 18F25.