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PERTURBATION AND ANALYTIC CONTINUATION OF GROUP REPRESENTATIONS

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ABSTRACT. I introduce a theory of noncommutative bounded perturbations of Lie algebras of unbounded operators. When applied to group representations, it leads to an analytic embedding of the dual object of some semisimple Lie groups into the bounded operators on corresponding Hilbert spaces of K-finite vectors.

1. **Introduction.** I announce a general theorem on analytic continuation of group representations which is based on perturbation theory for linear operators. This result is a contribution of the author to a series of joint results with R. T. Moore reported in detail in [3]. Applications of the theorem to quasi-simple Banach representations of $SL(2, \mathbf{R})$, due to Moore, will be announced separately by him. The theorem introduces a perturbation theory for representations of Lie groups which generalizes the classical perturbation theory (due to R. S. Phillips [2, p. 389]) for one-parameter (semi) groups of bounded linear operators on a Banach space. Let $\{\pi(t): -\infty < t < \infty\}$ be such a strongly continuous one-parameter group (C_0 group) acting on a Banach space E. Let A be the infinitesimal generator of π, and let U be a "small" (bounded, say) perturbation of A, $B = A + U$. Then B generates a C_0 group $\{\pi_U(t)\}$ on E, and this group depends analytically on U (in a sense which is specified in [2, p. 404]). In my theorem the real line $\mathbf{R}$ is replaced by a Lie group G, and A is replaced by a Lie algebra L of unbounded operators in E. U is going to be a tuple $(U_1, \ldots, U_r)$ of bounded operators. In that way I obtain a surprisingly simple analytic continuation picture for a wide class of induced representations, and other unitary and nonunitary representations.

2. **Assumptions.** I first restrict the class of perturbations U to be considered. In order to make sure that π_U is a representation of the same group for all U, I assume that the corresponding infinitesimal operator Lie algebras L_U are all algebraically isomorphic.

Let D be a linear space. Let $\mathfrak{A}(D)$ be the algebra of linear endomorphisms of D. It is also a real Lie algebra when equipped with the commutator bracket, $[A, B] = AB - BA$ for $A, B \in \mathfrak{A}(D)$. The Lie algebra L generated by a subset S of $\mathfrak{A}(D)$ is defined to be the smallest *real* Lie subalgebra of $\mathfrak{A}(D)$ which contains

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