

CLASSIFICATION OF SIMPLICIAL TRIANGULATIONS OF TOPOLOGICAL MANIFOLDS

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In this note we announce theorems which classify simplicial (not necessarily combinatorial) triangulations of a given topological n -manifold M , $n \geq 7$ (≥ 6 if $\partial M = \emptyset$), in terms of homotopy classes of lifts of the classifying map $\tau: M \rightarrow BTOP$ for the stable topological tangent bundle of M to a classifying space $BTRI_n$ which we introduce below. The (homotopic) fiber of the natural map $j: BTRI_n \rightarrow BTOP$ is described in terms of certain groups of PL homology 3-spheres. We also give necessary and sufficient conditions for a closed topological n -manifold M , $n \geq 6$, to possess a simplicial triangulation.

The proofs of these results incorporate recent geometric results of F. Ancel and J. Cannon [1], J. Cannon [2], R. D. Edwards [4], and D. Galewski and R. Stern [5].

In [8], R. Kirby and L. Siebenmann show that in each dimension greater than four there exist closed topological manifolds which admit no piecewise linear manifold structure and hence cannot be triangulated as a combinatorial manifold. Also, R. D. Edwards [3] has recently shown that the double suspension of the Mazur homology 3-sphere is homeomorphic to S^5 , thus showing that a simplicial triangulation of a topological manifold *need not* be combinatorial. But it is still unknown whether or not every topological manifold can be triangulated as a simplicial complex.

Our classification theorems for simplicial triangulations on a given topological manifold take the following forms:

Let $BTOP$ denote the classifying space for stable topological block bundles.

THEOREM 1. *There is a space $BTRI_n$ and a natural map $BTRI_n \rightarrow BTOP$ such that if M is a topological n -manifold, $n \geq 7$ (≥ 6 if $\partial M = \emptyset$) and $\tau: M \rightarrow BTOP$ classifies the stable topological tangent bundle of M , then there is a one-to-one correspondence between the set of concordance classes of simplicial triangulations of M and the set of vertical homotopy classes of lifts of τ to $BTRI_n$.*

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