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THE RATIONAL HOMOTOPY OF FIXED POINT SETS OF TORUS ACTIONS

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1. **Introduction.** Let X be a connected topological space, whose Sullivan-de Rham minimal model, $M(X)$, is finitely generated. Following Halperin [8], we shall denote the indecomposable quotient of $M(X)$ by $\Pi_\psi^*(X)$, and call it the pseudo-dual rational homotopy of X. If X is simply-connected, then $\Pi_\psi^n(X)$ is naturally isomorphic to $(\pi_n(X) \otimes Q)^*$, for all $n \geq 1$. (See [4] and [8] for detailed treatment of $\Pi_\psi^*(X)$.)

DEFINITION 1.1. If $\dim_Q \Pi_\psi^*(X) < \infty$, then we shall say that X has finite dimensional rational homotopy (FDRH), and we shall define the Euler-Poincaré homotopy characteristic of X to be $\chi\pi(X) = \Sigma_{n=1}^\infty (-1)^n \dim_Q \Pi_\psi^n(X)$.

In this note we announce some results, which relate $\Pi_\psi^*(X)$ to $\Pi_\psi^*(F)$, where F is a component of the fixed point set of a torus group action on X. Further results and detailed proofs will appear in [2] and [3].

2. **Results.** Although more general conditions would suffice, we shall assume, for simplicity, throughout this section, that X is a compact topological manifold, that a torus T is acting on X locally smoothly (that is, with linear slices), and that the fixed point set, X^T, is nonempty. Our first theorem is the following.

THEOREM 2.1. *If X has FDRH, and if F is a component of X^T, then F has FDRH, and $\chi\pi(F) = \chi\pi(X)$. Furthermore,*

$$\text{(i)} \qquad \sum_{n=1}^{\infty} \dim_Q \Pi_\psi^{2n}(F) \leq \sum_{n=1}^{\infty} \dim_Q \Pi_\psi^{2n}(X);$$

and

$$\text{(ii)} \qquad \sum_{n=0}^{\infty} \dim_Q \Pi_\psi^{2n+1}(F) \leq \sum_{n=0}^{\infty} \dim_Q \Pi_\psi^{2n+1}(X).$$

We also have the following generalization of Bredon's inequalities [5].

THEOREM 2.2. *If X has FDRH, then, for all $n \geq 1$,*

$$\dim_Q \Pi_\psi^n(F) \leq \sum_{k=0}^{\infty} \dim_Q \Pi_\psi^{n+2k}(X).$$

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