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Deterministic and stochastic optimal control, by Wendell H. Fleming and Raymond W. Rishel, Applications of Mathematics, vol. 1, Springer-Verlag, New York, Heidelberg and Berlin, 1975, 222 pp., $24.80.

The deterministic optimal control problem under consideration is the following. The state of a system evolves according to a system of first order ordinary differential equations whose right hand side is under our "control" and it is required to control the system so as to minimize a given functional of the control and state. This functional is usually called a performance index, following the engineering literature. In the stochastic optimal control problem the state is a finite dimensional diffusion which evolves according to a system of stochastic differential equations under our control, and it is required to control the system so as to minimize the expected value of a given performance index. The first half of the book deals with the deterministic problem; the second half deals with the stochastic problem.

The mathematical theory of deterministic optimal control is in a relatively complete and satisfactory state. The authors have focused their attention on three important and related aspects of this theory, which we shall discuss below. The stochastic theory is currently not as complete. The approach in this book is by way of dynamic programming. Much of the material was originally developed by the authors themselves and appears in book form for the first time.

There is considerable difference in the mathematical prerequisites for reading the two parts of the book. The first part should be accessible to anyone who has completed the standard first year graduate course in analysis. After a stage-setting initial chapter on the calculus of variations the authors focus on their three principal topics: the Pontryagin Maximum Principle (necessary conditions); existence theorems; and dynamic programming in relation to control problems. In their proof of the maximum principle the authors combine features of several different proofs to produce a very appealing proof of the theorem. The proofs of the existence theorems are essentially the original proofs of Filippov in the case of bounded controls and of Cesari in the case of unbounded controls. The dynamic programming chapter introduces some of the ideas used in the stochastic control problem and presents a sufficiency theorem in terms of the structure of the trajectory fields that one gets by "solving" the maximum principle. In the proof of the sufficiency theorem the specialist will appreciate the simplification achieved by the use of Federer's co-area formula.

The minimum mathematical prerequisite for part two is a beginning graduate course in probability which includes martingales. Knowledge of stochastic processes would also be useful. In Chapter V, which with the exception of one section is independent of the rest of the book, the authors give a crash course on those aspects of stochastic processes and related topics