interchange of ideas with physics. Nowadays, physicists are more interested in indefinite Riemannian metrics. Again, the authors' narrow viewpoint has precluded anything on this topic. In fact, in parallel with this work–and with almost no interdisciplinary communication–physicists interested in applications of general relativity to cosmology and astrophysics have used similar techniques to prove singularity theorems about nonpositive Riemannian metrics. See the book, *The large-scale structure of space-time*, by Hawking and Ellis.

This book is a historical landmark in the sense that it is the first to concentrate on the successes of post-World War II differential geometry. Every book published before has been more-or-less an attempt to understand the work of the great masters in the light of the modern sensibility. Above all else, we have had to struggle to understand Elie Cartan! (One can even trace the spirit of this book back to Cartan, particularly *Géométrie des éspaces de Riemann*.) As I have already mentioned, the great successes recounted here have been achieved at the expense of partially, and perhaps only temporarily, abandoning the sweeping outlook of the classical work. One has only to compare this material to that in the collected works of Cartan and Lie and in Darboux' *Théorie des surfaces* to realize how much of our heritage has been dumped overboard. Perhaps this is due to our overemphasis on maintaining our status in the eyes of our big brothers, the topologists. I recall that when I was a student in the fifties everyone almost went around chanting, in Red Guard fashion: global good, local bad. Of course, this fanaticism had the happy consequence of leading to this impressive work; one will never know what might have been achieved if differential geometry had kept its traditional orientation. I would now want to ask how the techniques so precisely and powerfully developed in this modern work can be applied to the broader classical problems. My own guess is that most likely the field awaiting conquest is the geometric theory of nonlinear partial differential equations. (For example, it is not at all well known that Darboux' formidable treatise contains much more about this subject than it does about the theory of surfaces!) Perhaps the seeds to great advances in this field–and the recent discovery of "solitons" suggests that it is also of great interest for physics–lie in Darboux just as the seeds that grew to these magnificent comparison theorems were buried in the work of Jacobi, Riemann and Ricci.

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Discrete-parameter martingales, by Jacques Neveu, North-Holland, Amsterdam; American Elsevier, New York, 1975, viii + 236 pp., $26.95.

It is very simple to define a martingale. If $\mathfrak{T}_1 \subset \mathfrak{T}_2 \subset \dots$ is an increasing sequence of sub σ-fields of the σ-field $\mathfrak{T}$ in a probability space $(\Omega, \mathfrak{T}, P)$, a sequence $\{x_n\}$ of real random variables is called *adapted* if x_n is $\mathfrak{T}_n$ measurable for $n = 1, 2, \dots$. The adapted sequence is further called a supermartingale if $E(x_{n+1}|\mathfrak{T}_n) \leqslant x_n$, $n \geqslant 1$. It is called a martingale if the inequality is replaced by an equality and a submartingale if the inequality is reversed. Just one more definition is effectively all that is needed. A positive integer