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Comparison theorems in Riemannian geometry, by Jeff Cheeger and David G. Ebin, North-Holland Mathematical Library, vol. 9, North-Holland, Amsterdam; American Elsevier, New York, 1975, viii + 174 pp., $20.95.

Differential geometry is an almost unique area within mathematics, since it involves both the old and the new in an essential way. Riemannian geometry itself has, of course, been around for over one hundred years: About twenty-five years ago geometers began to ask how the local curvature of Riemannian manifolds could influence their global properties. (There were clues that this was an interesting question, e.g., the theorem of Hadamard and Cartan that a complete simply connected Riemannian manifold of nonpositive sectional curvature was diffeomorphic to Euclidean space.) The major opening salvo in this campaign was Rauch's work, published in 1951, showing that a (positive definite) Riemannian manifold whose sectional curvature function is sufficiently close to the curvature of the usual metric on the sphere is, in fact, homeomorphic to the sphere. Rauch combined techniques whose roots lie in the classical work: Sturm-type theorems for the systems of linear ordinary differential equations which result from linearization of the geodesic equations, and the distance minimizing property of the geodesics. Berger, Klingenberg and Toponogov then developed the conditions on the curvature which assure that the manifold is homeomorphic to the sphere and analyzed what happens at the precise point that the conditions are violated. They also developed a refined and powerful methodology to deal with this type of problem. In the sixties the methods were successfully applied to two general problems: Find Rauch-type conditions on the curvature which would assure that the manifold is diffeomorphic to the sphere, and study general global