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GENERALIZED ZETA-FUNCTIONS FOR AXIOM A BASIC SETS

BY D. RUELLE

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Let X be a set, $f\colon X \longmapsto X$ a map, $\varphi\colon X \longmapsto \mathbf{C}$ a complex-valued function. We write formally

$$D(\varphi) = \exp\left[- \sum_{n=1}^{\infty} \frac{1}{n} \sum_{\xi \in \mathrm{Fix} f^n} \prod_{k=0}^{n-1} \varphi(f^k \xi) \right]$$

Taking φ constant, i.e. replacing φ by $z \in \mathbf{C}$, we can interpret $1/D(z)$ as a zeta-function proved to be rational for Axiom A diffeomorphisms by Guckenheimer and Manning [6].

Similarly, if (f^t) is a flow on X, we write formally

$$d(A) = \prod_{\gamma} \left[1 - \exp \int_0^{\lambda(\gamma)} A(f^t x_\gamma)\, dt \right]$$

where the product extends over the periodic orbits γ of the flow, $\lambda(\gamma)$ is the prime period of γ and x_γ a point of γ.

In this note we indicate analyticity properties of $A \longrightarrow D(e^A)$ or $A \longrightarrow d(A)$ for diffeomorphisms or flows satisfying Smale's Axiom A, assuming only that A is Hölder continuous. Our results hold in particular for Anosov diffeomorphisms and flows, and when A is C^1. Stronger properties of meromorphy hold under suitable assumptions of real-analyticity and will be published elsewhere by P. Cartier and the author.

Let Λ be a basic set for a C^1 diffeomorphisms or flow satisfying Smale's Axiom A (see [13]). Choosing a Riemann metric d, and $\alpha \in (0, 1)$ we let C^α be the Banach space of real Hölder continuous functions of exponent α, with the norm

$$\|A\|_\alpha = \sup \left\{ |A(x)| + \frac{|A(y) - A(x)|}{(d(x, y))^\alpha} : x, y \in \Lambda \text{ and } x \neq y \right\}$$

We denote by $C^\alpha_{\mathbf{C}}$ the corresponding space of complex functions.

1. THEOREM. *Let the Axiom* A *diffeomorphism f restricted to the basic set Λ be topologically mixing. We denote by $P(A)$ the (topological) pressure of a real continuous function A on Λ (see* [8], [14], [4]). *There is a continuous real function R on $C^\alpha_{\mathbf{C}}$ satisfying*

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