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EXTENSIONS OF C^*-ALGEBRAS AND ESSENTIALLY n-NORMAL OPERATORS

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Let H be a separable, infinite dimensional, complex Hilbert space, and let $L(H)$ be the algebra of all (bounded, linear) operators on H. The ideal of all compact operators on H will be denoted $K(H)$, and the (Calkin) quotient algebra $L(H)/K(H)$ will be denoted by $Q(H)$. Given a C^*-algebra A with identity, an extension τ of $K(H)$ by A (or simply an extension τ by A) is, by definition, an identity preserving injective *-homomorphism $\tau: A \longrightarrow Q(H)$. In [2] a complete classification of all extensions of $K(H)$ by any abelian separable C^*-algebra (with the natural equivalence relation) was obtained. As indicated in [2] and [3], if one wishes to attack the classification problem for extensions by noncommutative C^*-algebras, it is reasonable to restrict attention to separable ones. Henceforth, A will be assumed to be a separable C^*-algebra with identity. Also, we shall denote by π the canonical quotient map from $L(H)$ onto $Q(H)$. An extension τ by A will be said to be trivial if there exists a faithful nondegenerate *-representation $\sigma: A \longrightarrow L(H)$ such that $\tau = \pi\sigma$. It readily follows that trivial extensions by A always exist. We shall say that two extensions τ_1 and τ_2 by A are *equivalent* and we write $\tau_1 \approx \tau_2$ if there exists an operator W in $L(H)$ such that πW is a unitary element of $Q(H)$ and $\tau_1 A \pi W = \pi W \tau_2 A$, for every A in A. (In the terminology of [2] τ_1 and τ_2 are called weakly equivalent.) The set of all equivalence classes of extensions by A, under this equivalence relation, will be denoted by Ext A. Following the pattern of [2] and [3] we define a binary operation on Ext A as follows: let τ_1 and τ_2 be two extensions by A and let $\tau': A \longrightarrow Q(H) \oplus Q(H)$ given by $\tau' = \tau_1 \oplus \tau_2$; after identifying $Q(H) \oplus Q(H)$ with a C^*-subalgebra of $Q(H)$, we then obtain an extension τ by A whose equivalence class $[\tau]$ will be called the sum of $[\tau_1]$ and $[\tau_2]$.

The following theorem generalizes [2, Theorem 9.2].

THEOREM 1. *If every irreducible *-representation of A is finite dimensional, then* Ext A *is an abelian semigroup whose identity is the equivalence class of all trivial extensions by A. Moreover, if A also satisfies the property that for every identity preserving completely positive map $\varphi: A \longrightarrow Q(H)$, there exists an identity preserving completely positive map $\psi: A \longrightarrow L(H)$ such that $\varphi = \pi\psi$, then* Ext A *is a group.*

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