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LOCAL SURGERY AND APPLICATIONS TO THE THEORY OF QUADRATIC FORMS

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Let A be a unitary ring with involution and let $L_n^h(A)$ denote the surgery obstruction group in dimension n, defined by C. T. C. Wall in Chapters 5 and 6 of [13], for surgery to a homotopy equivalence. In this note a new local surgery theory [8], [9] is used to produce a localization sequence for Wall's L^h-groups (cf. Example 2 following Theorem 1). This sequence, together with a Mayer-Vietoris sequence derived from Sharpe's unitary periodicity for KU_i, $0 \leq i \leq 2$, can be used to make computations which include and extend many of the results of Bak [1], Bass [2], Karoubi [6], and Wall [14]. The approach we outline to the determination of surgery obstructions should be more accessible to topologists than other recent treatments, because the analysis involves only two nontrivial but essentially geometric tools: the localization sequence and the Mayer-Vietoris sequence. The first can be realized geometrically in the case of group rings; the second involves a formal construction in algebraic K-theory, together with Sharpe's study of the unitary Steinberg group, which can also be realized geometrically [11].

The localization sequence. Let A be a ring with involution and $\overline{A}$ a classical ring of quotients with respect to a multiplicative subset $\Sigma \subseteq A$. Five terms (starting with $L_{2k+1}^h(A)$) of the following localization sequence have been obtained by Karoubi [6] independently.

THEOREM 1. *Let A be a ring with involution, $\overline{A}$ a ring of quotients as above. Then there exists a sequence of groups $L_n^t(A, \Sigma)$ and a long exact sequence*

$$\cdots \longrightarrow L_{n+1}^h(\overline{A}) \longrightarrow L_n^t(A, \Sigma) \longrightarrow L_n^h(A) \longrightarrow L_n^h(\overline{A}) \longrightarrow L_{n-1}^t(A, \Sigma) \longrightarrow \cdots.$$

EXAMPLES. Let $\overline{A}$ denote the injective hull of A. In the examples below, $\overline{A}$ is a classical ring of quotients.

(1) If D is a Dedekind domain, $\overline{D} = F$ its fraction field, and the involution is trivial, then Theorem 1 (modified to replace L^h-groups with U-groups of [10, §3]) gives the Milnor exact sequence of [5],

$$0 \longrightarrow W(D) \longrightarrow W(F) \longrightarrow \coprod_p W(D/p),$$

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