BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 82, Number 1, January 1976

A PARTIAL TOPOLOGICAL CLASSIFICATION FOR STABLE MAP GERMS

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Communicated April 30, 1975

1. **Introduction.** The two theories of C^0 and C^∞-stability for smooth functions $f: M \longrightarrow N$ between smooth manifolds [2], [3], [5] both provide for appropriate dimensions a classification for a dense subset in the space of smooth mappings. In this note we announce a result which partially describes how distinct C^∞-stable map germs $f: \mathbf{R}^n \longrightarrow \mathbf{R}^p$ with $n \leqslant p$ are related under the weaker notion of topological equivalence. We say after [1] that a stable map germ f is of *discrete algebra type* if there are only a finite number of germ types nearby with associated algebra an algebra in the same number of generators as $Q(f)$ (i.e. have same Σ_i type). This is the largest class of stable map germs which do not require moduli for their classification. It includes not only the stable map germs in the nice dimensions, but also simple stable map germs (in the same sense as used by Arnold for functions).

Following a conversation with Andre Galligo, it became clear that the best way to describe this partial topological classification is to use the Hilbert-Samuel function of the associated algebra.

THEOREM 1. *For stable map germs of discrete algebra type f, the Hilbert-Samuel function of $Q(f)$ is a topological invariant.*

The topological classification actually gives a stronger result for a number of cases, namely, that the complex algebra type is a topological invariant. The author hopes to complete this result in a subsequent paper.

Lastly, the author wishes to thank both Andre Galligo and John Mather.

2. **Germs of discrete algebra type.** These types were essentially determined by Mather [2–VI]. He determined where moduli first appeared in each Σ_i type except one. We recall that if $f: \mathbf{R}^n \longrightarrow \mathbf{R}^p$ is of type Σ_i at 0, then letting $K = \ker D_0 f$, $C = \operatorname{coker} D_0 f$, we have $\dim K = i$; and there is a second intrinsic derivative $\widetilde{D}_0^2 f: S^2K \longrightarrow C$. Then, f is of type $\Sigma_{i,(j)}$ if $\dim \ker(\widetilde{D}_0^2 f) = j$. If $\ker(\widetilde{D}_0^2 f) = S^2K$, then we can define $\widetilde{D}_0^3 f: S^3K \longrightarrow C$. We can repeat this until $\ker D_0^l f \neq S^l K$. This

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AMS (MOS) subject classifications (1970). Primary 57D45; Secondary 58C25, 57D35.
Key words and phrases. Stable map germs, topological invariance, Hilbert-Samuel function, discrete algebra type.

[1] Partially supported by a grant from the City University of New York Faculty Research Award Program.