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ON THE SUBALGEBRAS OF CERTAIN FINITELY PRESENTED ALGEBRAS

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1. Every finitely generated metabelian group can be embedded in a finitely presented metabelian group [2]. The object of this note is to announce some theorems of a similar nature for lie and associative algebras. Since associative algebras are structurally richer than groups, it is not surprising that the associative algebra versions of these theorems (see §2) encompass somewhat more than the corresponding known theorems for groups (see [5], [2], and [8]). The same cannot, however, be said at this time for lie algebras (see §3).

2. All of our embedding theorems for associative algebras stem from a simple criterion, Theorem A (below), for an associative algebra to be finitely presented. In order to explain, we need to introduce some notation and definitions. Let then T be an associative algebra over (as always) a commutative field. If n is any positive integer, then T^n is the subalgebra of T generated by all n-fold products of elements of T; notice that T^n is an ideal of T. T is termed nilpotent if $T^n = 0$ for some n. Now let M be a (T, T)-bimodule. We term M *ample* if for each $m \in M$, $t \in T$, there exist $t', t'' \in T$ such that $mt = t'm$, $tm = mt''$. The following theorem then holds.

THEOREM A. *Let* $0 \to N \to A \to T \to 0$ *be a short exact sequence of associative algebras, where N is nilpotent and all of the quotients of T are finitely presented. If N^i/N^{i+1}, viewed as a (T, T)-bimodule, is finitely generated and ample for every i, then A is finitely presented.*

On appealing to a theorem of J. Lewin [7], Theorem A can be used to deduce

THEOREM B. *Let A be a finitely generated associative algebra containing an ideal N with $N^2 = 0$. If every quotient of A/N is finitely presented then A can be embedded in a finitely presented algebra A^* which closely resembles A; specifically A^* contains an ideal N^* such that* (i) $N^{*2} = 0$ *and* (ii) $A^*/N^* = A/N \otimes A/N$.

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