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CONJUGATE SYSTEM CHARACTERIZATIONS OF H^1: COUNTER EXAMPLES FOR THE EUCLIDEAN PLANE AND LOCAL FIELDS

BY A. GANDULFO, J. GARCIA-CUERVA AND M. TAIBLESON[1]

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ABSTRACT. The characterization of the Hardy space, H^1 of the plane, as those integrable functions whose first order Riesz transforms are (or whose maximal function is) integrable is well known. J.-A. Chao and M. Taibleson have shown that there is a conjugate system characterization of H^1 of a local field that parallels the Riesz system characterization of $H^1(R^2)$. C. Fefferman has conjectured that "nice" conjugate systems, such as the second order Riesz transforms would also give a characterization of $H^1(R^2)$. In the present paper a counter example of A. Gandulfo and M. Taibleson is described that shows that any conjugate system generated by an even kernel will fail to characterize H^1 of a local field. A counter example of J. Garcia-Cuerva is described that shows that the second order Riesz system for the Euclidean plane (which is generated by an even kernel) will fail to characterize $H^1(R^2)$ in the above sense.

Let $f \in L^1(\mathbf{R}^n)$ and let $f^*(x) = \sup_{y>0} |f(x, y)|$, where $f(x, y)$ is the Poisson integral of f. We say that $f \in H^1(\mathbf{R}^n)$ iff $f^* \in L^1(\mathbf{R}^n)$. Let (r, θ) be the polar representation of $(x_1, x_2) \in \mathbf{R}^2$, and let $(\cdot)^\wedge$ and $(\cdot)^\vee$ represents the Fourier transform and its inverse. The following characterization of $H^1(\mathbf{R}^2)$ is in [5, §8]:

THEOREM A. *If f is real-valued and $f \in L^1(\mathbf{R}^2)$, then $f \in H^1(\mathbf{R}^2)$ iff $(e^{i\theta}\hat{f})^\vee \in L^1(\mathbf{R}^2)$.*

Similarly, if K is a local field, e.g., a p-adic field, we may define $f^*(x) = \sup_{k\in \mathbf{Z}} |f(x, k)|$, where $f(x, k)$ is the regularization of f. (See [6, Chapter IV].) We say that $f \in H^1(K)$ iff $f^* \in L^1(K)$. The following characterization of $H^1(K)$ follows from results of Chao and Taibleson [3] and Chao [1], [2].

THEOREM B. *Suppose π is a multiplicative character on K that is unitary, ramified of degree 1, homogeneous of degree 0 and odd. If $f \in L^1(K)$ then $f \in H^1(K)$ iff $(\pi\hat{f})^\vee \in L^1(K)$.*

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