BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 82, Number 1, January 1976

ON THE NUMBER OF SOLUTIONS TO PLATEAU'S PROBLEM

BY A. J. TROMBA[1]

Communicated by S. S. Chern, August 28, 1975

Introduction. Since its formulation by Plateau in the 19th century, little (see [2], [4]) has been known about the number of simply connected minimal surfaces spanning a simple closed curve $\Gamma \subset R^3$. Existence was proved in the thirties by J. Douglas [1] and T. Radó [5]. In the paragraphs below we indicate how a new topological theory partially describes the way in which the number of minimal surfaces spanning a curve changes as the curve changes.

I. **Formulation of the problem.** Let $H^{r+2}(S^1, R^n)$ be the Sobolev Hilbert space of H^{r+2} maps the unit circle S^1 into R^n, with $r \geqslant 5$. Let $\mathcal{A} = \text{Emb}(S^1, R^3)$ be the open submanifold of $H^{r+2}(S^1, R^3)$ which consists of embeddings of S^1 into R^3. Let Γ be the image of such an embedding $\alpha \in \mathcal{A}$. Set η^α to be the component of $H^2(S^1, \Gamma)$ {the C^r Hilbert manifold of H^2 maps from S^1 to Γ} determined by the embedding α. Let $\mathcal{M}^\alpha$ be the open submanifold of η^α consisting of the diffeomorphisms. For every $u \in H^2(S^1, \Gamma) \subset H^2(S^1, R^3)$ we can extend $u = (u_1, \ldots, u_n)$ harmonically to the disc $\mathcal{D}$. Define the smooth energy functional $E_\alpha: \eta^\alpha \longrightarrow R$ by

$$E_\alpha(u) = \frac{1}{2} \sum_{i=1}^{3} \int_{\mathcal{D}} \left[\left(\frac{\partial u_i}{\partial x} \right)^2 + \left(\frac{\partial u_i}{\partial y} \right)^2 \right] dx\, dy.$$

Denote by $\overline{\mathcal{M}}^\alpha$ the closure of $\mathcal{M}^\alpha$ in η^α.

J. Douglas showed, in his pioneering work [1], that the critical points of E_α in $\overline{\mathcal{M}}^\alpha$ are simply connected minimal surfaces spanning Γ. We are interested in obtaining information on the number of critical points of E_α on $\overline{\mathcal{M}}^\alpha$.

II. **The theory.** Let M be a connected smooth Banach manifold and $K: T^2M \longrightarrow TM$ a connection map. In [6] the author defines a smooth vector field $X: M \longrightarrow TM$ to be Fredholm with respect to K if for each $p \in M$ the covariant derivative of X with respect to K, $\nabla X(p)$, which is a linear map of T_pM to itself, is linear Fredholm. By the *index of X* we mean the dim ker $\nabla X(p)$ − dim coker $\nabla X(p)$. A Fredholm vector field is Palais-Smale if $\nabla X(p)$ is of the form $I + C$, where C is a completely continuous linear map. Palais-Smale vector fields have index zero.

AMS (MOS) subject classifications (1970). Primary 35G20, 49F10, 58E15; Secondary 57D25.

[1]Research partially supported by National Science Foundation grants GP-39060 and MPS72–05055 A02.