

INVARIANCE PRINCIPLE FOR MODIFIED WAVE OPERATORS

BY COLSTON CHANDLER AND A. G. GIBSON¹

Communicated by Chandler Davis, July 19, 1975

1. **Introduction.** The invariance principle of Birman and Kato (see e.g. [5]) states that, for simple scattering systems with short-range potentials, the wave operator limits

$$(1) \quad \Omega_{\pm}(\phi(H_2), \phi(H_1)) \equiv \text{s-lim}_{t \rightarrow \pm\infty} e^{it\phi(H_2)} e^{-it\phi(H_1)} P_1$$

are independent of ϕ for a wide class of functions, and equal, respectively, the wave operators

$$(2) \quad \Omega_{\pm}(H_2, H_1) \equiv \text{s-lim}_{t \rightarrow \pm\infty} e^{itH_2} e^{-itH_1} P_1.$$

Kato first proved the invariance under the assumption that $H_2 - H_1$ is a trace-class operator. It has since been proved under some alternative assumptions on H_1 and H_2 (see e.g. [4], [6]).

For other scattering systems, such as scattering with long-range potentials, the limits (1) and (2) may not exist; however, certain modified wave operators (see (3) and (4) below) may exist [1], [3]. An invariance principle for modified wave operators has been proved by Matveev [6], [7] and Sakhnovich [8] under certain rate-of-convergence assumptions. However, these assumptions are shown to be satisfied only for a class of short-range potentials [6, Theorem 2].

In this note we announce the result that the invariance principle of scattering theory is valid in practically all situations in which (possibly modified) time-dependent wave operators are known to exist.

2. **Notation.** Let H_k be selfadjoint operators on separable Hilbert spaces H_k , $k = 1, 2$. Let P_1 denote the orthogonal projection of H_1 onto the space $H_{1,\text{ac}}$ of absolute continuity for H_1 . Let Δ be some closed and bounded interval of the real axis \mathbf{R} , and let $E_1(\Delta)$ be the corresponding spectral projection of the operator H_1 . Let \mathcal{D}_1 be the dense subset of vectors $u \in H_{1,\text{ac}}$ with $\|u\| < \infty$,

AMS (MOS) subject classifications (1970). Primary 47A40, 81A45; Secondary 35J10, 42A68, 47F05.

Key words and phrases. Scattering theory, wave operators, invariance principle, Schrödinger operator, Fourier transform.

¹The authors were supported in part by Sandia Laboratories, SURP contract 51-6640.

Copyright © 1975, American Mathematical Society