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A COMPUTING METHOD FOR BIFURCATION BOUGHS OF NONLINEAR EIGENVALUE PROBLEMS

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Bifurcation branches of solutions of nonlinear (differential) equations dependent on a real-parameter is now a well developed theory. (See [4] for general aspects, [3] for existence and [1] for examples.) However, what is known is mostly bifurcate solutions in a neighbourhood of the bifurcation points, and there exist few numerical methods that allow us to obtain these branches.

The method we propose allows us to obtain the "regular" parts of these branches, that means without "critical" points, as solutions of a Cauchy problem in which the real bifurcation parameter is the variable.

1. **Definitions.** Let X and Z be two real-Banach spaces and f a continuously Fréchet-differentiable mapping from $X \times \mathbf{R}$ to Z. $D_1 f$ denotes the partial derivative in x. We seek solutions (x, y) in $X \times \mathbf{R}$ of the following equation:

$$f(x, y) = 0; \tag{1}$$

y is often called the bifurcation parameter. Let $\mathcal{S}$ be the set of these solutions and $\mathcal{C}$ the subset of $\mathcal{S}$ for which $D_1 f(x, y)$ is not a homeomorphism ($\mathcal{C}$ is the set of "critical solutions"); then every maximal (with respect to the relation of inclusion) connected subset $\mathcal{B}$ of $\mathcal{S} - \mathcal{C}$ is called bifurcation bough.

2. **One property of bifurcation boughs.** We shall use the following classical result (Lemma 1) to prove the main theorem.

LEMMA 1. *Let E be a separate, connected, topological space, and h a local homeomorphism from E to* $\mathbf{R}$; *then h is a homeomorphism from E onto $h(E)$.*

The theorem which gives the main property of bifurcation boughs is:

THEOREM 1. *For every bifurcation bough $\mathcal{B}$, let $\mathcal{I}$ (resp. $\mathcal{K}$) be its projection on* $\mathbf{R}$ *(resp. X). There exists a unique continuously differentiable mapping Ψ from $\mathcal{I}$ to $\mathcal{K}$ such that:*

(i) *The graph of Ψ in $X \times \mathbf{R}$ is $\mathcal{B}$.*

(ii) $\Psi'(y) = -[D_1 f(\Psi(y), y)]^{-1} \circ D_2 f(\Psi(y), y)$.

PROOF. At every point (x, y) of $\mathcal{B}$, we can apply the implicit function theorem; there exists an open interval $I_y \subset \mathbf{R}$ including y, and a unique mapping φ

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