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STABILITY OF EQUIVARIANT SMOOTH MAPS

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This research announcement is a summary of a paper which will appear elsewhere [5], and which continues the program started in [4].

1. We consider a compact Lie group G and smooth compact G-manifolds X and Y. By $C_G^\infty(X, Y)$, $\mathrm{Diff}_G(X)$, $\mathrm{Diff}_G(Y)$ we denote the C^∞, *G-equivariant* mappings $X \longrightarrow Y$, respectively, diffeos of X or diffeos of Y.

There is a natural group action

$$\mathrm{Diff}_G(X) \times \mathrm{Diff}_G(Y) \times C_G^\infty(X, Y) \xrightarrow[\Phi]{} C_G^\infty(X, Y),$$

and for each $f \in C_G^\infty(X, Y)$, we define the corresponding orbit-map

$$\mathrm{Diff}_G(X) \times \mathrm{Diff}_G(Y) \xrightarrow[\Phi_f]{} C_G^\infty(X, Y).$$

We consider the G-bundles TX, TY, f^*TY and their "invariant sections" $\Gamma^\infty(TX)^G$, $\Gamma^\infty(TY)^G$, $\Gamma^\infty(f^*TY)^G$. (These are modules over the corresponding rings of G-invariant functions.)

As in the usual case [3], [6] we have linear mappings

$$\begin{array}{ccc} \Gamma^\infty(TX)^G & \searrow^{\beta_f} & \\ & & \Gamma^\infty(f^*TY)^G \\ \Gamma^\infty(TY)^G & \nearrow_{\alpha_f} & \end{array}$$

defined in a natural way.

By definition, f is infinitesimally stable if $\alpha_f + \beta_f$ is surjective.

By definition, f is stable if Image Φ_f is a neighbourhood of $f \in C_G^\infty(X, Y)$.

With these definitions we have the

STABILITY THEOREM. *Let $f \in C_G^\infty(X, Y)$ be infinitesimally stable. Then:*

(i) *Whenever Z_1 is the germ of a metrizable or compact topological space, Z_2 the germ of a smooth finite dimensional manifold, and $\psi: Z_1 \times Z_2 \longrightarrow C_G^\infty(X, Y)$ a $C^{0,\infty}$-germ of a map sending the base points to f, there is a germ of a $C^{0,\infty}$ map $\Psi: Z_1 \times Z_2 \longrightarrow \mathrm{Diff}_G(X) \times \mathrm{Diff}_G(Y)$ sending the base points to* $(\mathrm{id}\ X) \times (\mathrm{id}\ Y)$ *and such that*

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