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QUADRATIC FORMS AND SIMILARITIES

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1. **Introduction.** The results announced here are concerned with the Hurwitz problem of composition of quadratic forms, over a field F characteristic not two. The possible dimensions of forms admitting composition are stated. However, determining which quadratic forms do admit composition is a more delicate question, and answers are known only for small dimensions. The proofs will appear elsewhere.

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2. **The Hurwitz problem.** We follow the notation of Lam's book **[L]**. Throughout this paper, F will denote a field, with char $F \neq 2$. All forms considered will be nonsingular.

DEFINITION. If (V, q) is a quadratic space over F, a map $f \in \text{End}(V)$ is a *similarity* if

$$q(f(v)) = \sigma(f) \cdot q(v), \quad \text{for all } v \in V,$$

where $\sigma(f) \in F$. Let $\text{Sim}(V, q) = \text{Sim}(q)$ denote the set of all similarities on (V, q).

We are interested in the additive structure if $\text{Sim}(V, q)$. If S is an F-linear subspace of $\text{End}(V)$, and $S \subseteq \text{Sim}(V, q)$, then the map $\sigma\colon \text{Sim}(V, q) \longrightarrow F$ becomes a quadratic form when restricted to S. We consider only those subspaces S on which this form is nonsingular.

Notation. For quadratic forms σ, q, we write $\sigma < \text{Sim}(q)$ if σ is isometric to some subspace of $\text{Sim}(q)$, using the induced quadratic form.

Using the definition of composition of quadratic forms in **[L**, p. 133**]**, we see that q admits composition with σ if and only if $\sigma < \text{Sim}(q)$. The study of such composition began with the four and eight square problem, and was completed by Hurwitz in the case when F is algebraically closed **[H]**.

Notation. Following **[L]**, **[K2]**, for $a_i \in F$, we write $\langle a_1, \ldots, a_n\rangle$ for an n-dimensional diagonal form; and $\langle\langle a_1, \ldots, a_n\rangle\rangle$ for the n-fold Pfister form $\bigotimes_{i=1}^n \langle 1, a_i\rangle$. For quadratic forms φ, q, we write $\varphi \simeq q$ if they are isometric,

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