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OPERATOR ALGEBRAS AND ALGEBRAIC K-THEORY

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1. **Introduction.** We wish to announce several related results which demonstrate a relationship between operator theory and algebraic K-theory. Some of these results concern extensions of C^*-algebras (cf. [4], [5]) and complement the results of [4]. Others concern the trace and determinant invariants defined in [7].

2. **Extensions of C^*-algebras.** Let H be a separable infinite dimensional Hilbert space, $L(H)$ the algebra of bounded linear operators on H, K the ideal of compact operators, and $A = L(H)/K$. In [4] and [5] $\operatorname{Ext}(X)$ was defined as the set of equivalence classes of C^*-algebra extensions, $0 \to K \to E \to C(X) \to 0$, for X a compact metric space and $C(X)$ the algebra of continuous complex functions on X. $\operatorname{Ext}(X)$ was also described as unitary equivalence classes of $*$-isomorphisms τ: $C(X) \to A$. It was shown that $\operatorname{Ext}(X)$ is a group and that it gives rise to a generalized homology theory which is related to K-theory in roughly the same way as homology is related to cohomology. A Bott periodicity map, Per: $\operatorname{Ext}(S^2X) \to \operatorname{Ext}(X)$, was defined and was proved to be injective for all X and surjective for smooth X. Also $\operatorname{Ext}(X)$ was given the structure of a not necessarily Hausdorff topological group, and the closure of the identity was called $\operatorname{PExt}(X)$.

THEOREM 1. Per *is surjective for all X.*

THEOREM 2. *There is a natural short exact sequence,*

$$0 \to \operatorname{Ext}^1_{\mathbf{Z}}(K^0(X), \mathbf{Z}) \to \operatorname{Ext}(X) \xrightarrow{\gamma_\infty} \operatorname{Hom}(\widetilde{K}^1(X), \mathbf{Z}) \to 0,$$

which splits noncanonically.

COROLLARY. $\operatorname{PExt}(X)$ *is the maximum divisible subgroup of* $\operatorname{Ext}(X)$.

THEOREM 3. *If τ_t: $C(X) \to A$, $0 \leqslant t \leqslant 1$, is a continuous family in the sense that $\tau_t(f)$ is continuous for each $f \in C(X)$, then each τ_t defines the same element of* $\operatorname{Ext}(X)$.

For a more leisurely account of these results, see [3]. See also [4], [5], [8]. Ext_* satisfies parallel axioms to the Steenrod homology theory [11], whose axiomatic description in [10] plays a key role in the proofs. Algebraic K-theory

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