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DUALITY FOR CROSSED PRODUCTS OF VON NEUMANN ALGEBRAS BY LOCALLY COMPACT GROUPS

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The duality for crossed products of von Neumann algebras by locally compact abelian groups has been obtained by Takesaki [4]. We shall generalize this result to a locally compact (not necessarily abelian) group by using the Fourier algebra in place of the dual group.

Let G denote a locally compact group with a right invariant Haar measure dt, and M denote a von Neumann algebra over a Hilbert space $\mathcal{H}$. By an *action* of G on M we mean a homomorphism σ: $t \in G \mapsto \sigma_t \in \mathrm{Aut}(M)$ such that for each $x \in M$ the mapping $t \in G \mapsto \sigma_t(x)$ is σ-strongly* continuous. Let $\{\pi_\sigma, \lambda\}$ be a covariant representation of $\{M, \sigma\}$ on $\mathcal{H} \otimes L^2(G)$ defined by

$$\begin{cases} (\pi_\sigma(x)\xi)(s) \equiv \sigma_s(x)\xi(s), & \xi \in \mathcal{H} \otimes L^2(G), \\ \lambda(r)\xi(s) \equiv \xi(sr), & r, s \in G. \end{cases}$$

Then the crossed product $\mathcal{R}(M; \pi_\sigma)$ of M by G is the von Neumann algebra generated by $\pi_\sigma(M)$ and $\lambda(G)$.

THEOREM 1. *A necessary and sufficient condition that a mapping α of M into $M \otimes L^\infty(G)$ be induced by an action σ with*

$$(\alpha(x)\xi)(s) = \sigma_s(x)\xi(s), \qquad x \in M,\ \xi \in \mathcal{H} \otimes L^2(G),$$

is that α be an isomorphism with the commutative diagram:

$$\begin{array}{ccc} M & \xrightarrow{\quad\alpha\quad} & M \otimes L^\infty(G) \\ {\scriptstyle\alpha}\big\downarrow & & \big\downarrow{\scriptstyle\alpha \otimes \iota} \\ M \otimes L^\infty(G) & \xrightarrow{\iota \otimes \delta} & M \otimes L^\infty(G) \otimes L^\infty(G), \end{array} \tag{1}$$

where $(\delta f)(s, t) \equiv f(st)$ for $f \in L^\infty(G)$.

For the right regular representation λ_G of G on $L^2(G)$, i.e.,

$$(\lambda_G(s)f)(t) \equiv f(ts), \qquad f \in L^2(G),\ s, t \in G,$$

let $R(G)$ denote the von Neumann algebra generated by $\lambda_G(G)$. Let γ denote the isomorphism of $R(G)$ into $R(G) \otimes R(G)$ defined by

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