BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 81, Number 6, November 1975

# ADJOINT SEMIGROUP THEORY FOR A VOLTERRA INTEGRODIFFERENTIAL SYSTEM

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Communicated by Richard Miller, June 23, 1975

I. **Introduction.** In this note we announce some recent results concerning the semigroup theory for a class of linear Volterra integrodifferential systems. The system under consideration has previously been studied by Barbu and Grossman [2] and Miller [6], via semigroup methods. Although semigroup theory is employed in both of the above mentioned articles, it is important to note that the semigroup constructed in [6] differs greatly from the semigroup constructed in [2]. In particular, Miller is able to obtain certain stability results that do not hold for the semigroup constructed in [2]. However, we show that by an appropriate choice of the state space, Miller's semigroup may be considered as the "adjoint" semigroup (in the sense of Hille and Phillips [5]) to the semigroup constructed by Barbu and Grossman. We shall state the results without proofs. Proofs of the theorems will appear elsewhere (see [3]).

II. **Preliminaries.** If $x$: $(-\infty, 0] \longrightarrow C^n$ is given, then for $t \geqslant 0$, we define $x_t$: $(-\infty, 0] \longrightarrow C^n$ by $x_t(s) = x(t+s)$. For $1 \leqslant p \leqslant +\infty$, the usual Lebesgue space of $C^n$-valued functions on an interval with endpoints $-\infty \leqslant a < b \leqslant +\infty$ will be denoted by $L_p(a, b)$. Throughout this paper, $M$ shall denote an $n \times n$ constant matrix and $K(\cdot)$ shall denote an $n \times n$ matrix function satisfying $\int_0^{+\infty} \|K(s)\|\, ds < +\infty$. Consider the linear Volterra integrodifferential equation,

$$x'(t) = Mx(t) + \int_{-\infty}^{t} K(t-s)x(s)\, ds, \tag{2.1}$$

with the initial data

$$x(0) = \eta, \qquad x_0(s) = \varphi(s) \quad \text{a.e. on } (-\infty, 0], \tag{2.2}$$

where $\eta \in C^n$ and $\varphi \in L_1(-\infty, 0)$.

A solution to system (2.1)–(2.2) is a function $x$: $(-\infty, +\infty) \longrightarrow C^n$ such that $x$ is absolutely continuous (A.C.) on $[0, +\infty)$ and satisfies (2.1) a.e. on $[0, +\infty)$, $x(0) = \eta$, and $x_0(s) = \varphi(s)$ a.e. on $(-\infty, 0]$. We shall let $Z_1 = C^n \times L_1(-\infty, 0)$ denote the product space with the product norm. It can be shown

*AMS (MOS) subject classifications* (1970). Primary 45A05, 45D05.

*Key words and phrases.* Volterra integrodifferential system, semigroup, adjoint semigroup.