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AN L^1-SPACE FOR BOOLEAN ALGEBRAS AND SEMIREFLEXIVITY OF $L^\infty(X, \Sigma, \mu)$

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This note indicates how one can use the ideas of strict topologies on spaces of continuous functions to, at a single stroke, obtain an extended construction of L^1-spaces without reference to measure, obtain ordinary $L^1(X, \Sigma, \mu)$-spaces as the natural dual of $L^\infty(X, \Sigma, \mu)$ and obtain a view of the dual pairing (L^∞, L^1) that is very much like that of (C, M), where C is a space of bounded continuous functions and M a space of bounded Baire or Borel measures.

Earlier results, [1] and [4], suggest this development. In [1], Buck shows that M, the compact regular Borel measures on locally compact X, results as $(C(X), \beta)'$, where β is the topology on $C(X)$ defined by the seminorms $\|f\|_\xi = \sup\{|f(x)\xi(x)|: x \in X\}$, with $\xi \in C$ vanishing at ∞. In [4], this writer showed how β-methods extend to completely regular X, with $\xi \equiv 0$ over compact sets, or zero sets, in $\beta X \backslash X$. For $X = \{1, 2, \ldots\}$, $l^\infty = C$ and $l^1 = M$, and by [1], $(l^\infty, \beta)' = l^1$. By choosing $\xi \equiv 0$ over certain closed nowhere dense subsets of the appropriate Stone space, we show herein that this result is more than the small coincidence formally expected.

2. **The space $L^1(A)$.** Let A be a Boolean algebra [6] and let S be its Stone space with $\eta(a) \subset S$ denoting the compact-open set corresponding to $a \in A$.

We define an indicator function on A, $\chi: A \longrightarrow C(S)$, by $\chi(a) = \chi_{\eta(a)}$ and let $L^\infty(A)$ be the closed linear span of $\chi(A)$ in $C(S)$ in the $\|\ \|$ (=uniform convergence on S) topology on $C(S)$. In fact, $L^\infty(A) = C(S)$.

For each increasing sequence $a_n \in A$ with $a = \sup a_n$ (i.e., $a_n \nearrow a$), let $Q = \eta(a) \backslash \bigcup_{n=1}^\infty \eta(a_n)$ and define the β_Q topology on $L^\infty = L^\infty(A)$ by the seminorms $\|f\|_\xi$ for $f \in L^\infty$ where $\xi \in C(S)$ and $\xi \equiv 0$ on Q. Let β be the inductive limit topology over all such Q. We remark that β may be neither Hausdorff, nor finer than pointwise convergence and is the $\|\ \|$-topology iff all increasing $\{a_n\}$ with a supremum in A are finite.

We now define $L^1(A)$ by $L^1(A) = (L^\infty(A), \beta)'$, the β-dual of $L^\infty(A)$. It is possible that $L^1(A) = \{0\}$ ([6, p. 65] and (2) below).

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