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THE STRUCTURE OF SINGULARITIES IN AREA-RELATED VARIATIONAL PROBLEMS WITH CONSTRAINTS

BY JEAN E. TAYLOR[1]

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This is a research announcement of results whose full details and proofs have been submitted for publication elsewhere. We provide a complete description, both combinatorial and differential, of the local structure of singularities in a large class of two-dimensional surfaces in $\mathbf{R}^3$, those which are $(\mathbf{M}, \epsilon, \delta)$ minimal **[TJ1]** and those which are $(\mathbf{F}, \epsilon, \delta)$ minimal for a Hölder continuous ellipsoidal integrand F **[TJ2]**. Such surfaces include mathematical models for compound soap bubbles **[AF1]**, **[AF2]** and soap films, thereby settling a problem which has been studied for well over a century (a very general formulation of Plateau's Problem); in general, $(\mathbf{M}, \epsilon, \delta)$ and $(\mathbf{F}, \epsilon, \delta)$ minimal surfaces arise as solutions to geometric variational problems with constraints.

$(\mathbf{M}, \epsilon, \delta)$ and $(\mathbf{F}, \epsilon, \delta)$ minimal surfaces were defined, shown to exist, and proven to be regular almost everywhere in **[AF2]** (see **[AF1]** for a brief description). We define $Y \subset \mathbf{R}^3$ as the union of the half disk $\{x \in \mathbf{R}^3: x_1^2 + x_2^2 \leqslant 1, x_2 \geqslant 0, x_3 = 0\}$ with its rotations by 120° and 240° about the x_1 axis, and define $T \subset \mathbf{R}^3$ as $C \cap \{x: |x| \leqslant 1\}$, where C is the central cone over the one-skeleton of the regular tetrahedron centered at the origin and containing as vertices the points $(3, 0, 0)$ and $(-1, 2\sqrt{2}, 0)$. Varifold tangents are defined in **[AW 3.4]** and a tangent cone is defined to be the support of a varifold tangent.

The major result of **[TJ1]** is the following.

THEOREM. *Suppose S is $(\mathbf{M}, \epsilon, \delta)$ minimal with respect to some closed set B, where $\epsilon(r) = Cr^\alpha$ for some $C < \infty$ and $\alpha > 0$. Then*

(1) *there exists a unique tangent cone, denoted* $\mathrm{Tan}(S, p)$, *to S at each point p in S,*

(2) $R(S) = \{p \in S$: $\mathrm{Tan}(S, p)$ *is a disk*$\}$ *is a two-dimensional Hölder continuously differentiable submanifold of* $\mathbf{R}^3$, *with* $\mathcal{H}^2(R(S)) = \mathcal{H}^2(S)$ **[AF1]**, **[AF2]** (*here* $\mathcal{H}^2$ *denotes* (*Hausdorff*) *two-dimensional area*),

(3) $\sigma_Y(S) = \{p \in S$: $\mathrm{Tan}(S, p) = \theta Y$ *for some θ in* $\mathbf{O}(3)$, *the group of*

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