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CLASSIFICATION OF AUTOMORPHISMS OF HYPERFINITE FACTORS OF TYPE II_1 AND II_∞ AND APPLICATION TO TYPE III FACTORS

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ABSTRACT. For each integer $p = 0, 1, 2 \cdots$ and complex number γ, $\gamma^p = 1$ ($\gamma = 1$ for $p = 0$) we define an automorphism s_p^γ of the hyperfinite factor of type II_1, R. For any automorphism α of R there is a unique couple (p, γ) and a unitary $v \in R$ such that α is conjugate to $\mathrm{Ad}\, v \circ s_p^\gamma$. Let $R_{0,1}$ be the tensor product of R by a I_∞ factor. There is, up to conjugacy, only one automorphism θ_λ of $R_{0,1}$ such that θ_λ multiplies the trace by λ, provided $\lambda \neq 1$.

Introduction. The classification of type III factors that we proposed in [2] relates isomorphism classes of type III_λ factors, $\lambda \in]0, 1[$ with outer conjugacy classes of automorphisms of factors of type II_∞. An obvious criticism to the value of such a relation is then the following: Is it possible to classify automorphisms even for the simplest factor of type II_∞, namely $R_{0,1}$ the tensor product of R, the hyperfinite II_1, by a I_∞ factor. We answer this question in this paper, showing that for any $\lambda \in]0, 1[$ there is only one automorphism, up to conjugacy, of $R_{0,1}$ which multiplies the trace by λ. The proof of this fact relies on the classification of automorphisms of the hyperfinite factor R (see Theorem 1) which in turn uses mainly the analogy between classical ergodic theory and ergodic theory on a nonabelian von Neumann algebra.

Automorphisms of the hyperfinite factor of type II_1. Recall that if M is a factor and $\theta \in \mathrm{Aut}\, M$, one defines the outer period $p_0(\theta)$ as the period of θ modulo inner automorphisms (i.e. $\theta^k \in \mathrm{Int}\, M \Leftrightarrow k \in p_0(\theta)Z$). Also the obstruction of θ, noted $\gamma(\theta)$, is the root of unity in $\mathbf{C}$ such that ($\theta^{p_0} = \mathrm{Ad}\, v$, v unitary in M) $\Rightarrow \theta(v) = \gamma v$. Finally α and $\beta \in \mathrm{Aut}\, M$ are outer conjugate iff β is conjugate to the product of α by an inner automorphism.

THEOREM 1. *Two automorphisms α, β of R are outer conjugate if and only if $p_0(\alpha) = p_0(\beta)$ and $\gamma(\alpha) = \gamma(\beta)$.*

In particular, any two aperiodic automorphisms α, β of R are outer conjugate. This relies on an analogue of Rokhlin's theorem. In the case $p_0(\alpha) \neq 0$ the proof uses the tensor product as a group structure on the set $Br(Z/p, R)$ of

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