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JOINT SPECTRUM IN THE CALKIN ALGEBRA

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For a nice discussion pertaining to the essential spectrum of a single operator (bounded linear transformation) in a complex separable infinite dimensional Hilbert space H, the reader is referred to Fillmore, Stampfli and Williams [4]. The purpose of this note is to announce analogous results concerning the joint essential spectra of n-tuples of operators in H.

Joint essential spectrum. In the sequel $L(H)$ denotes the algebra of all operators on H and K denotes the ideal of compact operators on H. Let ν be the canonical homomorphism from $L(H)$ onto the *Calkin algebra* $L(H)/K = C$. If $A = (A_1, \ldots, A_n)$ is an n-tuple of operators on H, then we write $\nu(A_j) = a_j$, the coset containing A_j for each j, $1 \leq j \leq n$, and $a = (a_1, \ldots, a_n)$.

The *joint essential spectrum* of an n-tuple of operators A denoted by $\sigma_e(A)$ is defined to be the *joint spectrum* $\sigma(a)$ of a.

Here $\sigma(a) = \sigma^l(a) \cup \sigma^r(a)$, where the *left* (*right*) *joint spectrum* $\sigma^l(a)$ ($\sigma^r(a)$) is defined as the set of all $z = (z_1, \ldots, z_n)$ in $\mathbf{C}^n$ (n-fold Cartesian product of the set of all complex numbers $\mathbf{C}$) such that $\{a_j - z_j\}_{1 \leq j \leq n}$ generates a proper left (right) ideal in the Calkin algebra C. For this notion of joint spectrum, the reader may consult [1] and [5]. We call the set $\sigma^l(a)$ ($\sigma^r(a)$) as the *left* (*right*) *joint essential spectrum* and denote it by $\sigma_e^l(A)$ ($\sigma_e^r(A)$). Clearly, $\sigma_e^l(A) \subseteq \sigma^l(A)$, $\sigma_e^r(A) \subseteq \sigma^r(A)$; and hence $\sigma_e(A) \subseteq \sigma(A)$. Further, if $A = (A_1, \ldots, A_n)$ is an n-tuple of *essentially commuting* (commuting modulo the compacts) operators, then $\sigma_e(A)$ is a nonempty compact subset of $\mathbf{C}^n$.

The following theorem describes the relationship between the joint spectrum and the joint essential spectrum of an n-tuple of operators.

THEOREM 1. *Let $A = (A_1, \ldots, A_n)$ be an n-tuple of operators on H. Then $\sigma(A) = \sigma_e(A) \cup \sigma_p(A) \cup \sigma_p(A^*)^*$, where $A^* = (A_1^*, \ldots, A_n^*)$ and star on the right represents complex conjugates.*

A point $z = (z_1, \ldots, z_n)$ of $\mathbf{C}^n$ is in $\sigma_p(A)$ (the *joint eigenvalue* of A) if

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