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NOVIKOV'S EXT^2 AND THE NONTRIVIALITY OF THE GAMMA FAMILY

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Larry Smith [7] defined and detected elements β_t in the p-primary component of the stable homotopy of the sphere for $t > 0$ and $p \geq 5$. In the same manner, Toda's construction [11] gives elements γ_t for $t > 0$ and $p \geq 7$. We have the following results which are a consequence of our computation of the second line of the Novikov spectral sequence for a sphere at an odd prime.

THEOREM 1. (a) *p does not divide* $\beta_t \in \pi^S_{2(p^2-1)t-2(p-1)-2}(S^0)$ *for* $p \geq 5$, $t > 0$.

(b) $0 \neq \gamma_t \in \pi^S_{2(p^3-1)t-2(p^2-1)-2(p-1)-3}(S^0)$ *for* $p \geq 7, t > 0$.

(c) $\alpha_1\beta_t \neq 0$ *for* $t \neq 0$ *or* -1 mod p, $p \geq 5$.

Partial results on the nontriviality of γ_t have been obtained by Thomas and Zahler [10], [9], Oka and Toda [6], Johnson, Miller, Wilson, and Zahler [2], and Ravenel (unpublished).

These infinite families can be studied most conveniently by means of the Novikov spectral sequence

$$E_2^{**} = \mathrm{Ext}^{**}_{BP_*BP}(BP_*, BP_*(X)) \Rightarrow \pi_*(X)_{(p)}$$

for a space X [1]. $BP_*(\ )$ is the Brown-Peterson homology theory [1], and

$$BP_* = BP_*(S^0) = \mathbf{Z}_{(p)}[v_1, v_2, \ldots], \qquad |v_i| = 2(p^i - 1).$$

Let I_n denote the invariant ideal $(p, v_1, \ldots, v_{n-1}) \subset BP_*$; $I_0 = (0)$. For a BP_*BP comodule M let H^*M denote $\mathrm{Ext}^{**}_{BP_*BP}(BP_*, M)$. By a theorem of Landweber [3] we have for $n > 0$

$$H^0BP_*/I_n = \mathbf{F}_p[v_n].$$

Let δ_n: $H^iBP_*/I_{n+1} \longrightarrow H^{i+1}BP_*/I_n$ be the connecting homomorphism in the long exact sequence associated with

$$0 \longrightarrow BP_*/I_n \xrightarrow{v_n} BP_*/I_n \longrightarrow BP_*/I_{n+1} \longrightarrow 0.$$

It is folklore (see [2]) that if $p \geq 7$, $t > 0$, and $0 \neq \delta_0\delta_1\delta_2(v_3^t) \in H^3BP_*$, then this class survives to γ_t and $\gamma_t \neq 0$. Our proof of Theorem 1 involves an analysis

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