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ON HOLOMORPHIC REPRESENTATIONS OF SYMPLECTIC GROUPS

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Let G denote the complex symplectic group which may be defined by the equation

$$G = \left\{ g \in \mathrm{GL}(2k, \mathbf{C}) \colon g s_k g^t = s_k,\ s_k = \begin{bmatrix} 0 & -I_k \\ I_k & 0 \end{bmatrix} \right\}.$$

In this paper we shall give a simple and concrete realization of a set of representatives of all irreducible holomorphic representations of G. This realization, which involves the G-module structure of a symmetric algebra of polynomial functions is inspired by the work of B. Kostant [1] and follows the general scheme formulated in [2]. Detailed proofs will appear elsewhere.

1. **The symmetric algebra $S(E^*)$.** Set $E = \mathbf{C}^{n \times 2k}$ with $k \geqslant n \geqslant 2$; then G acts linearly on E by right multiplication. Let $(\cdot\,,\cdot\,)$ denote the skew-symmetric bilinear form on E given by

$$(X, Y) = \mathrm{trace}(X s_k Y^t), \quad \forall X, Y \in E.$$

If $X \in E$, let X^* denote the linear form $Y \longrightarrow (X, Y)$ on E. The map $X \longrightarrow X^*$ establishes an isomorphism between E and its dual E^*. Let $S(E^*)$ denote the symmetric algebra of all complex-valued polynomial functions on E. The action of G on E induces a representation R of G on $S(E^*)$ defined by

$$(R(g)p)(X) = p(Xg), \quad \forall p \in S(E^*), \ \forall X \in E.$$

If $X \in E$, define a differential operator $X^*(D)$ on $S(E^*)$ by setting

$$(X^*(D) f)(Y) = \{(d/dt) f(Y + tX)\}_{t=0},$$

for all $f \in S(E^*)$, $t \in \mathbf{R}$, and $X, Y \in E$.

Define $(X_1^* \cdots X_n^*)(D)f = X_1^*(D)((X_2^* \cdots X_n^*)(D)f)$ inductively on n. If

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