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ABSOLUTE CONTINUITY AND WEAK COMPACTNESS

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In this paper we discuss a class of *a priori* inequalities of a type appearing frequently in linear partial differential equations and outline the connection with the class of all weakly compact operators. The point of departure is the well-known result due to Bartle, Dunford and Schwartz [1] asserting the existence of a control measure for each weakly compact operator given on a space $C(S)$ and the reader can remark easily that our main result (Theorem 4) provides a more precise estimate than that obtained in [1]. Estimates of the same type hold also for p-absolutely summing operators ($1 \leqslant p < \infty$) as well as for the compact ones, which shows an interesting connection between these classes. Finally we extend the well-known criterion of compactness in a space l_p ($1 \leqslant p < \infty$) by proving that, on a space which does not contain an isomorph of l_1, the compact operators are completely determined by certain *a priori* inequalities.

The details will appear elsewhere.

Let X, Y be two Banach spaces and let p be a continuous seminorm on X. The following definition extends considerably a basic concept in the theory of vector measures:

1. DEFINITION. An operator $T \in L(X, Y)$ is said to be *absolutely continuous* with respect to p (i.e., $T \ll p$) if the following equivalent conditions hold:

(AC_1) for every $\epsilon > 0$ there is a $\delta = \delta(\epsilon) > 0$ such that if $\|x\| \leqslant 1$, $p(x) < \delta$, then $\|Tx\| < \epsilon$;

(AC_2) for every $\epsilon > 0$ there is a $\delta = \delta(\epsilon) > 0$ such that $\|Tx\| \leqslant \epsilon\|x\| + \delta p(x)$ whenever $x \in X$;

(AC_3) given a bounded sequence $\{x_n\}_n \subset X$, then either there exists a positive constant $c > 0$ such that $\|T(x_n)\| \leqslant cp(x_n)$ for all $n \in \mathbf{N}$ or there exists a subsequence $\{x_{n_k}\}_k$ such that $T(x_{n_k}) \longrightarrow 0$.

This notion is implicit in many papers on partial differential equations in which case p is associated with an inner product. Inequalities such as Gårding's or Friedrichs' are consequences of the fact that suitable operators are absolutely continuous.

A local condition for absolute continuity was introduced in [5] for operators given on Banach lattices and used to describe the structure of Banach lattices

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